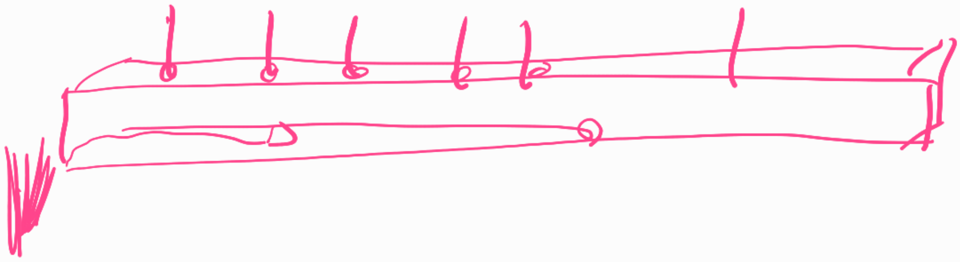
ADRIANO BARRA - PDE

$$\frac{\partial^2\varphi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2\varphi}{\partial t^2} = 0$$ Eq. D'alambert (1) ONDE, informazione

$$\frac{\partial^2 T}{\partial x^2} - k\frac{\partial T}{\partial t} = 0$$ Eq. Fourier (2) DISORDINE



$$F = -kx = m\frac{d^2x}{dt^2}$$

$t \to -t$

$$m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + k\cancel{x} = 0$$ oscillatore smorzato

os. aristotelico

$$m\frac{d^2x}{dt^2} + kx = 0$$ oscillatore armonico

os. platonico

$$m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = 0$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$

$t \to -t$

$\beta > 0$

$R > 0$

PISTONICO

$F = -kx \quad U = -\partial_x F = -\partial_x\left(\frac{1}{2}kx^2\right) = -kx$

$m\ddot{x} + k = 0 \qquad \ddot{x} = -\omega^2 x \qquad \omega = \sqrt{\frac{k}{m}} \qquad \omega^2 = \frac{k}{m}$

$x(t) = A\cos(\omega t + \varphi)$

$\dot{x}(t) = -A\omega\sin(\omega t + \varphi)$

$\ddot{x}(t) = -A\omega^2\cos(\omega t + \varphi)$

$\boxed{\ddot{x} = -\omega^2 x(t)}$

$E = T + U$

$T = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}\cancel{m}A^2\frac{k}{\cancel{m}}\sin^2(\omega t + \varphi)$

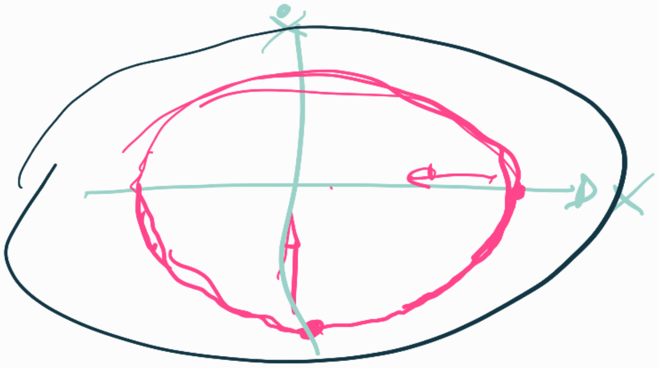
$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \varphi)$

$E = T + U = \frac{1}{2}kA^2\left[\underbrace{\sin^2(\omega t + \varphi) + \cos^2(\omega t + \varphi)}_{=1}\right]$

↳ $E = \frac{1}{2} k A^2 \neq f(t)$

$m\ddot{x} = -kx$

$mgh \rightarrow \frac{1}{2} m v^2$



$$\ddot{x} + \omega_s \dot{x} + \omega_0 x = 0 \qquad \omega_s = \frac{B}{m} \wedge \omega_0 = \sqrt{k/m}$$

cerco ~~sol.~~ sol. $\underline{x(t) = e^{\lambda t}}$

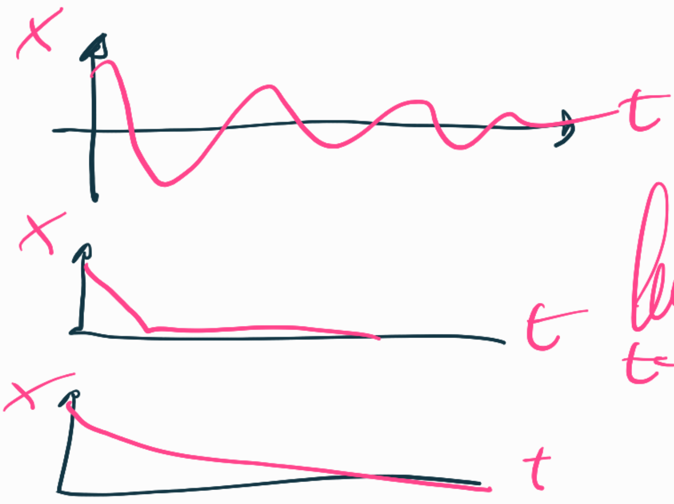
$$\dot{x} = \lambda e^{\lambda t} = \lambda x$$

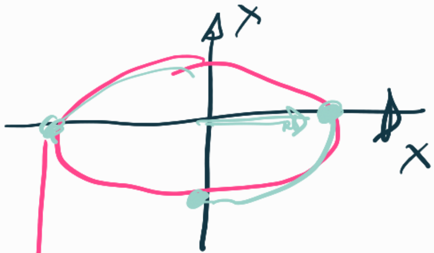
$$\ddot{x} = \lambda^2 e^{\lambda t} = \lambda^2 x$$

$$\boxed{\lambda^2 + \omega_s \lambda + \omega_0^2 = 0} \quad \begin{cases} \lambda_1 = -\frac{1}{2}(\omega_s + \sqrt{\Delta}) \\ \lambda_2 = -\frac{1}{2}(\omega_s - \sqrt{\Delta}) \end{cases}$$

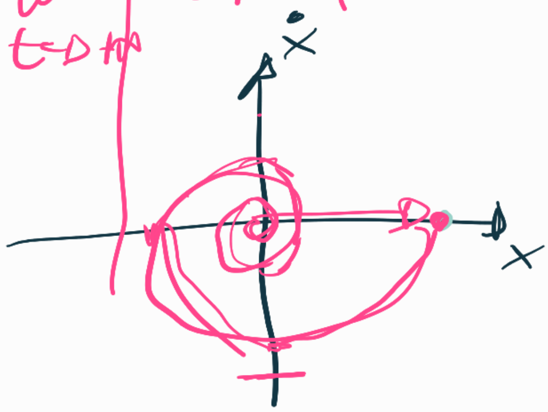
dove $\Delta = \omega_s^2 - 4\omega_0^2$

Δ:
- $\Delta < 0$
- $\Delta = 0$
- $\Delta > 0$





$$\lim_{t \to +\infty} x(t) = 0$$



ARISTOTELE $F = \text{cost} \to v = \text{cost}$

NEWTON $F = \text{cost} \to a = \text{cost}$

IL PARADOSSO ARISTOTELICO

$$mg - \beta v = ma = m\frac{dv}{dt}$$

$$\frac{dv}{mg - \beta v} = \cancel{\frac{1}{m}} = \frac{1}{m}dt$$

$mg - \beta$

$$\frac{dv}{mg - \beta v} = \frac{1}{m}dt$$

pongo $mg - \beta v = y$

$-\beta dv = dy \to dv = -\frac{1}{\beta}dy$

$$\boxed{\frac{dy}{y} = -\frac{\beta}{m}dt}$$

$$\int_{y(0)}^{y(t)} \frac{dy}{y} = -\frac{\beta}{m}\int_0^t dt \to \ln y\Big|_{y(0)}^{y(t)} = -\frac{\beta}{m}t$$

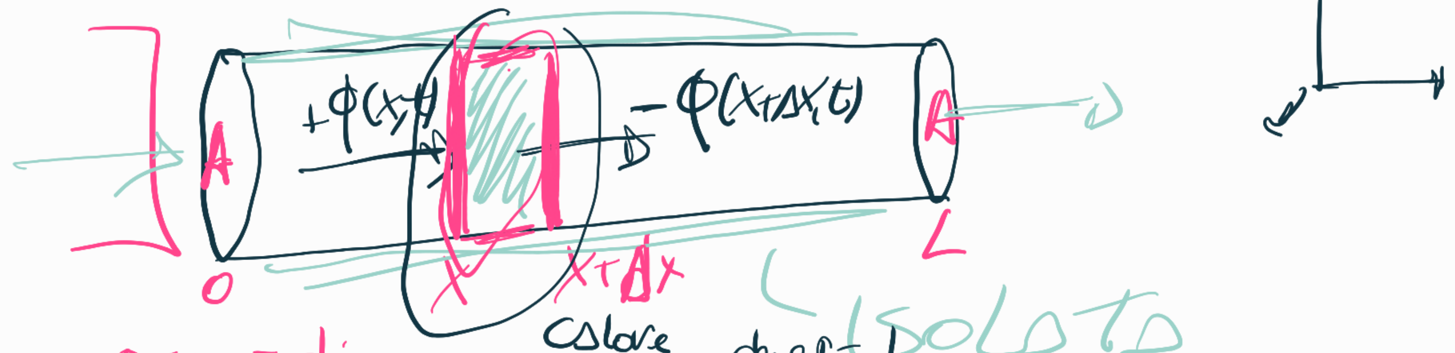
$$\ln y(t) - \ln y_0 = -\frac{\beta}{m}t$$

$$\ln\left[\frac{y(t)}{y_0}\right] = -\frac{\beta}{m}t \to \boxed{y(t) = y_0 e^{-\frac{\beta}{m}t}}$$

$$\lim_{t \to +\infty} y(t) = \emptyset \qquad 0 = mg - \beta v_L$$

$$\boxed{v_L = \frac{mg}{\beta}}$$

DERIVAZIONE della PDE di Fourier 1D



DENSITÀ di E TERMICA — calore specifico — densità di massa

def $$e(x,t) = c \cdot \rho \cdot u(x,t)$$

profilo temperatura

$$\boxed{E(x,t) = e(x,t)\, A \cdot \Delta x}$$

CONSERVAZIONE dell'energia termica

$$\underbrace{\text{Rate of change of Heat energy per time}} = \underbrace{\text{Heat energy flowing out of the boundaries} \times \text{UNIT TIME}} + \underbrace{\text{Heat energy generated inside} \times \text{unit time}}$$

$$\frac{\partial}{\partial t}E = \frac{\partial}{\partial t}\left[e(x,t)\cdot \cancel{A}\cdot \Delta x\right] = \left[\phi(x,t)\cdot A - \phi(x+\Delta x,t)\cdot \cancel{A}\right] + Q(x,t)\,\cancel{A}\cdot\Delta x$$

ϕ = Flusso di calore = la quantità di energia termica che fluisce lungo il tubo per unità di tempo e di area

$$\frac{\partial e}{\partial t} = \lim_{\Delta x \to 0} \left[\frac{\varphi(x,t) - \varphi(x+\Delta x, t)}{\Delta x} \right] + Q(x,t)$$

DERIVAZIONE DIFFERENZIALE

$$(1) \quad \boxed{\frac{\partial e}{\partial t} = -\frac{\partial \varphi}{\partial x} + Q}$$

WARNING:

INC: e, φ, q ? 1

$$\frac{d}{dt}\int_a^b e(x,t)\,dx = \underbrace{\varphi(a,t) - \varphi(b,t)}_{-\int_a^b \frac{\partial \varphi(x,t)}{\partial x}} + \int_a^b Q(x,t)\,dx$$

Approccio integrale

$$\int_a^b \left[\frac{\partial e}{\partial t} + \frac{\partial \varphi}{\partial x} - Q \right] dx = 0$$

$$= 0 \Rightarrow \boxed{\frac{\partial e}{\partial t} = -\frac{\partial \varphi}{\partial x} + Q} \quad (1)$$

$$e(x,t) = c\rho u(x,t) \quad (2)$$

$$\boxed{\varphi = -k_0 \frac{\partial u}{\partial x}} \quad (3)$$

→ Fourier T

→ Fick D

$$① + ② + ③ \Rightarrow \boxed{c\rho \frac{\partial u}{\partial t} = -\partial_x \left[-k_0 \frac{\partial u}{\partial x} \right] + Q}$$

1 PDE di Fourier $\rightarrow$ $$\boxed{c\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k_0 \frac{\partial u}{\partial x}\right) + Q}$$

PDE Fourier $$\boxed{c\rho \frac{\partial u}{\partial t} = k_0 \frac{\partial^2 u}{\partial x^2} + Q} \rightarrow K = \frac{k_0}{c\rho}$$

$$\boxed{\frac{\partial u}{\partial t} = K \frac{\partial^2 u}{\partial x^2} + Q}$$

~~EQ.~~ PDE Di FOURIER

$f(x)$ $\quad u(x, y, z, t)$

CI $\rightarrow$ "equivalente" di Cauchy x ODE $\quad u(x, t=0) = f(x)$

BC $\rightarrow$ $u(x=0) = T_1 \wedge u(x=L) = T_2$

BC $\rightarrow$ $\frac{\partial u}{\partial x}(x=0) = 0 \wedge \frac{\partial u}{\partial x}(x=L) = 0$ $\quad \phi = -K\frac{\partial u}{\partial x}$

SBARRA ISOLATO

ESERCIZIO: steady state distribution

$u(x,t) \longrightarrow u(x)$ sotto hp. SSD

$\boxed{BC}$ $\quad u(0,t) = T_1 \qquad u(L,t) = T_2$

$$\cancel{\frac{\partial U}{\partial t}} = K\frac{\partial^2 U}{\partial x^2} \qquad U(0,t)=T_1 \;,\; U(L,t)=T_2$$

$$K\frac{d^2 U}{dx^2}=0 \rightarrow \frac{d^2U}{dx^2}=0 \qquad \frac{dU}{dx}=C_1 \qquad \boxed{U(x)=C_1X+C_2}$$

$$U(0)=\boxed{T_1=C_2} \qquad \underline{U(L)=T_2=C_1L+T_1=T_2}$$

$$C_1L=T_2-T_1 \rightarrow C_1=\frac{T_2-T_1}{L}$$

$$\boxed{U(X)=\frac{(T_2-T_1)}{L}X+T_1}$$