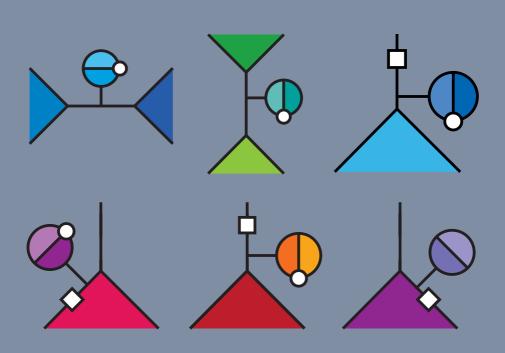
# Journal of Physics A

Mathematical and Theoretical

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# Highlights

A collection of outstanding research published in 2013





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# Welcome

#### **Martin Evans Editor-in-Chief**

Welcome to the Journal of Physics A: Mathematical and Theoretical Highlights collection. This collection showcases some of the most highly rated articles published in the journal over the course of 2013.

This selection of articles displays the broad scope of the journal and demonstrates how it is a meeting place for researchers to share mathematically rich work across different disciplines. Readers of the journal enjoy high-quality research articles from across the breadth of theoretical and mathematical physics.

Some important new discoveries in mathematical and theoretical physics have been published as Fast Track Communications (FTCs). FTCs are short timely articles that benefit from accelerated publication. Some of our most highly rated FTCs are included here.

Also included in this collection are details of the topical reviews published in 2013. These commissioned topical reviews provide timely overviews of the current state of research in areas of great interest and activity.

In 2013 we published three special issues, which all attracted high-quality papers from leaders in their fields. These special issues were on "Higher spin theories and holography", "Lyapunov analysis: from dynamical systems theory to applications" and "Logarithmic conformal field theory". Coming up in 2014 we have two special issues planned on "50 years of Bell's theorem" and "Cluster algebras in mathematical physics".

We hope that you will enjoy this celebration of highlighted articles.



# From the Publisher

#### Rebecca Gillan **Publisher**

We published some excellent papers in 2013. We'd like to thank all of our authors for choosing to submit their high-quality work to the journal, and thank our referees and board members for providing constructive peer review and maintaining the quality standards of the iournal.

We hope that you will find this collection of articles interesting and that you will consider Journal of Physics A for your next paper.

We look forward to working with you during 2014 and into 2015.

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Cover image: Exactly solvable lattice models with crossing symmetry Steven H Simon and Paul Fendley 2013 J. Phys. A: Math. Theor. 46 105002

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# Fast Track Communications



Université Paris Sud 11 and CNRS, France

# Large deviations of the top eigenvalue of large Cauchy random matrices

Satva N Maiumdar, Grégory Schehr, Dario Villamaina and Pierpaolo Vivo

2013 J. Phys. A: Math. Theor. 46 022001

We compute analytically the large deviation tails of the probability density function (pdf) of the top eigenvalue  $\lambda$ max in rotationally invariant and heavy-tailed Cauchy ensembles of N × N matrices for any Dyson index  $\beta > 0$ , where  $\beta = 1, 2, 4$  correspond, respectively, to orthogonal, unitary and symplectic ensembles. Furthermore, we show that these large deviation tails flank a central non-Gaussian regime for  $\lambda_{max} \sim \mathcal{O}_{(M)}$  on both sides. By matching these tails with the central regime, we obtain the exact leading asymptotic behaviors for any β of the pdf in the central regime, which generalizes the Tracy-Widom distribution known for Gaussian ensembles. Our analytical results are confirmed by numerical simulations.



Università di Salerno.

### Heat exchanges in a quenched ferromagnet

Federico Corberi, Giuseppe Gonnella, Antonio Piscitelli and Marco Zannetti

2013 J. Phys. A: Math. Theor. 46 042001

The off-equilibrium probability distribution of the heat exchanged by a ferromagnet in a time interval after a quench below the critical point is calculated analytically in the large-N limit. The distribution is characterized by a singular threshold QC < 0, below which a macroscopic fraction of heat is released by the k = 0 Fourier component of the order parameter. The mathematical structure producing this phenomenon is the same responsible for the order parameter condensation in the equilibrium low temperature phase. The heat exchanged by the individual Fourier modes follows a non-trivial pattern. with the unstable modes at small wave vectors warming up the modes around a characteristic finite wave vector kM. Two internal temperatures, associated with the k = 0 and k = kM modes, rule the heat currents through a fluctuation relation similar to the one for stationary systems in contact with two thermal reservoirs.

#### **FAST TRACK COMMUNICATIONS**



Giampaolo Cristadoro Università di Bologna Italy

# Follow the fugitive: an application of the method of images to open systems

#### G Cristadoro, G Knight and M Degli Esposti

2013 J. Phys. A: Math. Theor. 46 272001

Borrowing and extending the method of images we introduce a theoretical framework that greatly simplifies analytical and numerical investigations of the escape rate in open systems. As an example, we explicitly derive the exact size- and position-dependent escape rate in a Markov case for holes of finite-size. Moreover, a general relation between the transfer operators of the closed and corresponding open systems, together with the generating function of the probability of return to the hole is derived. This relation is then used to compute the small hole asymptotic behavior, in terms of readily calculable quantities. As an example we derive logarithmic corrections in the second order term. Being valid for Markov systems, our framework can find application in many areas of the physical sciences such as information theory, network theory, quantum Weyl law and, via Ulam's method, can be used as an approximation method in general dynamical systems.



Polish Academy of Sciences and Jagiellonian University, Poland

# Majorization entropic uncertainty relations

#### Zbigniew Puchała, Łukasz Rudnicki and Karol Życzkowski

2013 J. Phys. A: Math. Theor. 46 272002

Entropic uncertainty relations in a finite-dimensional Hilbert space are investigated. Making use of the majorization technique we derive explicit lower bounds for the sum of Rényi entropies describing probability distributions associated with a given pure state expanded in eigenbases of two observables. Obtained bounds are expressed in terms of the largest singular values of submatrices of the unitary rotation matrix. Numerical simulations show that for a generic unitary matrix of size N = 5, our bound is stronger than the well-known result of Maassen and Uffink (MU) with a probability larger than 98%. We also show that the bounds investigated are invariant under the dephasing and permutation operations. Finally, we derive a classical analogue of the MU uncertainty relation, which is formulated for stochastic transition matrices.

#### **FAST TRACK COMMUNICATIONS**



**Denis Bernard CNRS** and Ecole Normale Supérieure de Paris, France



**Beniamin Dovon** King's College London, UK

# Time-reversal symmetry and fluctuation relations in non-equilibrium quantum steady states

#### **Denis Bernard and Benjamin Doyon**

2013 J. Phys. A: Math. Theor. 46 372001

In this communication, we present a simple derivation, from time-reversal symmetry, of fluctuation relations for steady-state large deviation functions in non-equilibrium quantum systems. We further show that a condition of pure transmission implies extended fluctuation relations, connecting large deviation functions to mean currents at shifted temperatures and chemical potentials. We illustrate these concepts in various examples, including the interacting resonant level model and conformal or integrable models.



Michael V Berry University of Bristol,



Pragya Shukla Indian Institute of Science, Kharagpur, India

### Physical curl forces: dipole dynamics near optical vortices

#### M V Berry and Pragya Shukla

2013 J. Phys. A: Math. Theor. 46 422001

The force on a particle with complex electric polarizability is known to be not derivable from a potential, so its curl is non-zero. This 'curl force' is studied in detail for motion near an anisotropic optical vortex of arbitrary strength. Fundamental questions are raised by the fact that although the curl force requires the polarizability to have a non-zero imaginary part, reflecting absorption or scattering ('dissipation') in the internal dipole dynamics, the particle motion that it generates is non-dissipative (volume-preserving in the positionvelocity state space).

#### **FAST TRACK COMMUNICATIONS**



Rafael I Nenomechi University of Miami. USA

# An inhomogeneous T-Q equation for the open XXX chain with general boundary terms: completeness and arbitrary spin

#### Rafael I Nepomechie

2013 J. Phys. A: Math. Theor. 46 442002

An inhomogeneous T-O equation has recently been proposed by Cao, Yang, Shi and Wang for the open spin-1/2 XXX chain with general (nondiagonal) boundary terms. We argue that a simplified version of this equation describes all the eigenvalues of the transfer matrix of this model. We also propose a generating function for the inhomogeneous T-Q equations of arbitrary spin.



**David Reeb** Technische Universität München, Germany

### Coexistence does not imply joint measurability

#### David Reeb, Daniel Reitzner and Michael M Wolf

2013 J. Phys. A: Math. Theor. 46 462002

One of the hallmarks of quantum theory is the realization that distinct measurements cannot in general be performed simultaneously, in stark contrast to classical physics. In this context the notions of coexistence and joint measurability are employed to analyse the possibility of measuring together two general quantum observables, characterizing different degrees of compatibility between measurements. It is known that two jointly measurable observables are always coexistent, and that the converse holds for various classes of observables, including the case of observables with two outcomes. Here we resolve, in the negative, the open question of whether this equivalence holds in general. Our resolution strengthens the notions of coexistence and joint measurability by showing that both are robust against small imperfections in the measurement setups.

# Statistical physics



P Sollich King's College London, UK (Section Editor)

Statistical physics encompasses the theory of many interacting entities. Originally founded on the description of states of matter comprising atoms and molecules, the theory now extends to the description of active objects and driven systems.

The development of simple mathematical models to elucidate emergent macroscopic behaviour has gone hand in hand with the development of new mathematical techniques for their solution. Building on our understanding of equilibrium states and phase transitions, the grand challenge now lies in the description of nonequilibrium states and myriad associated out-of-equilibrium phenomena. Journal of Physics A is a leading vehicle for mathematical and theoretical progress in the field.

This selection of articles displays high-quality work in areas of current activity. In "Matrix ansatz for the fluctuations of the current in the ASEP with open boundaries" (Alexandre Lazarescu J. Phys. A: Math. Theor. 46 145003), a matrix product ansatz is presented that allows access to the exact statistics of the fluctuations of the macroscopic particle current for finite sizes, as well as the probabilities of configurations conditioned on the mean current.

Localization effects in 1D systems are also represented in this collection. V E Kravtsov and VI Yudson (*J. Phys. A: Math. Theor.* **46** 025001) consider the distribution function of the eigenfunction amplitude at the center-of-band anomaly in the one-dimensional Anderson model.

Directed polymers in quenched random potentials and the Kardar–Parisi–Zang equation remain areas of significant interest. In "Two-point free energy distribution function in (1+1) directed polymers" (Victor Dotsenko J. Phys. A: Math. Theor. 46 355001) the Bethe ansatz technique reveals the two-point free energy distribution function in (1+1) directed polymers with fixed end-point boundaries. This work is further developed in T Imamura, T Sasamoto and H Spohn J. Phys. A: Math. Theor. 46 355002.

Experiments on trapped ultra-cold atomic gases have prompted many studies on quenches of Hamiltonian parameters. Mario Collura and Pasquale Calabrese (J. Phys. A: Math. Theor. 46 175001) study out-of-equilibrium time evolution after a local guench connecting two anisotropic spin-1/2 XXZ Heisenberg open chains via an impurity bond. This out-of-equilibrium behaviour generalizes the results for the ground-state entanglement entropy of the model.

These and other contributions listed overleaf point to the vitality of the field and the continuing leading role of Journal of Physics A.

# Statistics of anomalously localized states at the center of band E = 0 in the one-dimensional Anderson localization model

#### V E Kravtsov and V I Yudson

#### 2013 J. Phys. A: Math. Theor. 46 025001

We consider the distribution function  $P(|\psi|2)$  of the eigenfunction amplitude at the center-of-band (E = 0) anomaly in the one-dimensional tight-binding chain with weak uncorrelated on-site disorder (the one-dimensional Anderson model). The special emphasis is on the probability of the anomalously localized states (ALS) with  $|\psi|$ 2 much larger than the inverse typical localization length IO. Using the recently found solution for the generating function  $\Phi$ an(u, phgr) we obtain the ALS probability distribution P( $|\psi|2$ ) at  $|\psi|$  210 Gt 1. As an auxiliary preliminary step, we found the asymptotic form of the generating function  $\Phi$ an(u, phgr) at u Gt 1 which can be used to compute other statistical properties at the center-of-band anomaly. We show that at moderately large values of  $|\psi|$  210, the probability of ALS at E = 0 is smaller than at energies away from the anomaly. However, at very large values of  $|\psi|$  210, the tendency is inverted: it is exponentially easier to create a very strongly localized state at E = 0 than at energies away from the anomaly. We also found the leading term in the behavior of  $P(|\psi|2)$  at small  $|\psi|2$  Lt I-10 and show that it is consistent with the exponential localization corresponding to the Lyapunov exponent found earlier by Kappus and Wegner.



Steven H Simon University of Oxford,



University of Virginia,

# Exactly solvable lattice models with crossing symmetry

#### **Steven H Simon and Paul Fendley**

#### 2013 J. Phys. A: Math. Theor. 46 095101

We show how to compute the exact partition function for lattice statistical—mechanical models whose Boltzmann weights obey a special 'crossing' symmetry. The crossing symmetry equates partition functions on different trivalent graphs, allowing a transformation to a graph where the partition function is easily computed. The simplest example is counting the number of nets without ends on the honeycomb lattice, including a weight per branching. Other examples include an Ising model on the Kagomé lattice with three-spin interactions, dimers on any graph of corner-sharing triangles, and noncrossing loops on the honeycomb lattice, where multiple loops on each edge are allowed. We give several methods for obtaining models with this crossing symmetry, one utilizing discrete groups and another anyon fusion rules. We also present results indicating that for models which deviate slightly from having crossing symmetry, a real-space decimation (renormalization-group-like) procedure restores the crossing symmetry.



CEA Saclay, France

# Matrix ansatz for the fluctuations of the current in the ASEP with open boundaries

#### Alexandre Lazarescu

2013 J. Phys. A: Math. Theor. 46 145003

The asymmetric simple exclusion process (ASEP) is one of the most extensively studied models in non-equilibrium statistical mechanics. The macroscopic particle current produced in its steady state is directly related to the breaking of detailed balance, and is therefore a physical quantity of particular interest. In this paper, we build a matrix product ansatz which allows us to access the exact statistics of the fluctuations of that current for finite sizes, as well as the probabilities of configurations conditioned on the mean current. We also show how this ansatz can be used for the periodic ASEP and how it translates into the language of the XXZ spin chain.



Université Paris-1, France

# Convex hull of n planar Brownian paths: an exact formula for the average number of edges

#### **Julien Randon-Furling**

2013 J. Phys. A: Math. Theor. 46 015004

We establish an exact formula for the average number of edges appearing on the boundary of the global convex hull of n independent Brownian paths in the plane. This requires the introduction of a counting criterion which amounts to 'cutting off' edges that are, in a specific sense, small. The main argument consists in a mapping between planar Brownian convex hulls and configurations of constrained, independent linear Brownian motions. This new formula is confirmed by retrieving an existing exact result on the average perimeter of the boundary of Brownian convex hulls in the plane.



Pasquale Calabrese Università di Pisa and INFN, Italy

# Entanglement evolution across defects in critical anisotropic Heisenberg chains

#### **Mario Collura and Pasquale Calabrese**

2013 J. Phys. A: Math. Theor. 46 175001

We study the out-of-equilibrium time evolution after a local guench connecting two anisotropic spin-1/2 XXZ Heisenberg open chains via an impurity bond. The dynamics is obtained by means of the adaptive time-dependent density-matrix renormalization group. We show that the entanglement entropies (von Neumann and Rényi) in the presence of a weakened bond depend on the sign of the bulk interaction. For an attractive interaction ( $\Delta$  < 0), the defect turns out to be irrelevant and the evolution is asymptotically equivalent to the one without defect obtained by conformal field theory. For a repulsive interaction  $(\Delta > 0)$ , the defect is relevant and the entanglement saturates to a finite value. This out-ofequilibrium behavior generalizes the well-known results for the ground-state entanglement entropy of the model.



Jean-Yves Fortin CNRS, France

### Dynamics of interval fragmentation and asymptotic distributions

#### Jean-Yves Fortin, Sophie Mantelli and MooYoung Choi

2013 J. Phys. A: Math. Theor. 46 225002

We study the general fragmentation process starting from one element of size unity (E = 1). At each elementary step, each existing element of size E can be fragmented into  $k \ge 2$ elements with probability pk. From the continuous time evolution equation, the size distribution function P(E; t) can be derived exactly in terms of the variable  $z = -\log E$ , with or without a source term that produces with rate r additional elements of unit size. Different cases are probed, in particular when the probability of breaking an element into k elements follows a power law:  $pk \propto k-1 - \eta$ . The asymptotic behavior of P(E; t) for small E (or large z) is determined according to the value of  $\eta$ . When  $\eta > 1$ , the distribution is asymptotically proportional to  $t^{1/4} \exp \left[ \sqrt{-\alpha t} \log E \right] \left[ -\log E \right]^{-3/4}$  with  $\alpha$  being a positive constant, whereas for  $\eta < 1$  it is proportional to  $E^{\eta - 1} t^{1/4} \exp \left[ \sqrt{-\alpha t \log E} \right] \left[ -\log E \right]^{-3/4}$  with additional timedependent corrections that are evaluated accurately with the saddle-point method.



Victor Dotsenko Université Paris VI. France and L D Landau Institute for Theoretical Physics, Russia

# Two-point free energy distribution function in (1+1) directed polymers

#### Victor Dotsenko

2013 J. Phys. A: Math. Theor. 46 355001

In this brief communication it is demonstrated how by using the Bethe ansatz technique the explicit expression for the two-point free energy distribution function in (1+1) directed polymers with fixed end-point boundary conditions can be derived in a rather simple way. The obtained result is equivalent to the one previously derived by Prolhac and Spohn (2011 J. Stat. Mech. P01031).



Takashi Imamura The University of Tokyo, Japan

# On the equal time two-point distribution of the one-dimensional KPZ equation by replica

#### T Imamura, T Sasamoto and H Spohn

2013 J. Phys. A: Math. Theor. 46 355002

In a recent contribution, Dotsenko establishes a Fredholm determinant formula for the two-point distribution of the Kardar-Parisi-Zhang equation in the long time limit and starting from narrow wedge initial conditions. We establish that his expression is identical to the Fredholm determinant resulting from the Airy2 process.



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# Fisher-Hartwig expansion for Toeplitz determinants and the spectrum of a single-particle reduced density matrix for one-dimensional free fermions

#### **Dmitri A Ivanov and Alexander G Abanov**

2013 J. Phys. A: Math. Theor. 46 375005

We study the spectrum of the Toeplitz matrix with a sine kernel, which corresponds to the single-particle reduced density matrix for free fermions on the one-dimensional lattice. For the spectral determinant of this matrix, a Fisher-Hartwig expansion in the inverse matrix size has been recently conjectured. This expansion can be verified order by order, away from the line of accumulation of zeros, using the recurrence relation known from the theory of discrete Painlevé equations. We perform such a verification to the tenth order and calculate the corresponding coefficients in the Fisher-Hartwig expansion. Under the assumption of the validity of the Fisher-Hartwig expansion in the whole range of the spectral parameter, we further derive expansions for an equation on the eigenvalues of this matrix and for the von Neumann entanglement entropy in the corresponding fermion problem. These analytical results are supported by a numerical example.



University of Edinburgh, UK

### An exclusion process on a tree with constant aggregate hopping rate

#### Peter Mottishaw, Bartlomiej Waclaw and Martin R Evans

2013 J. Phys. A: Math. Theor. 46 405003

We introduce a model of a totally asymmetric simple exclusion process (TASEP) on a tree network where the aggregate hopping rate is constant from level to level. With this choice for hopping rates the model shows the same phase diagram as the one-dimensional case. The potential applications of our model are in the area of distribution networks, where a single large source supplies material to a large number of small sinks via a hierarchical network. We show that mean-field theory (MFT) for our model is identical to that of the one-dimensional TASEP and that this MFT is exact for the TASEP on a tree in the limit of large branching ratio, b (or equivalently large coordination number). We then present an exact solution for the two level tree (or star network) that allows the computation of any correlation function and confirm how mean-field results are recovered as b  $\rightarrow \infty$ . As an example we compute the steady-state current as a function of branching ratio. We present simulation results that confirm these results and indicate that the convergence to MFT with large branching ratio is quite rapid.



**Anthony J Guttmann** The University of Melbourne, Australia

# A series test of the scaling limit of self-avoiding walks

#### Anthony J Guttmann and Jesper L Jacobsen

2013 J. Phys. A: Math. Theor. 46 435004

It is widely believed that the scaling limit of self-avoiding walks (SAWs) at the critical temperature is conformally invariant, and consequently describable by Schramm-Loewner evolution with parameter  $\kappa = 8/3$ . We consider SAWs in a rectangle, which originate at its centre and end at the boundary. We assume that the boundary density transforms covariantly in a way that depends precisely on  $\kappa$ , as conjectured by Lawler, Schramm and Werner. It has previously been shown by Guttmann and Kennedy that, in the limit of an infinitely large rectangle, the ratio of the fraction of SAWs hitting the side of the rectangle to the fraction that hit the end of the rectangle can be calculated. By considering rectangles of fixed aspect ratio 2, and also rectangles of aspect ratio 10, we calculate this ratio exactly for larger and larger rectangles. By extrapolating this data to infinite rectangle size, and invoking the above conjectures, we obtain the estimate  $\kappa = 2.66664 \pm 0.00007$  for rectangles of aspect ratio 2 and  $\kappa = 2.66675 \pm 0.00015$ for rectangles of aspect ratio 10. We also provide numerical evidence supporting the conjectured distribution of SAWs striking the boundary at various points in the case of rectangles with aspect ratio 2.



**Ralf Metzler** University of Potsdam, Germany and Tampere University of Technology, Finland

Correlated continuous time random walks: combining scaleinvariance with long-range memory for spatial and temporal dynamics

#### Johannes H P Schulz, Aleksei V Chechkin and Ralf Metzler

2013 J. Phys. A: Math. Theor. 46 475001

Standard continuous time random walk (CTRW) models are renewal processes in the sense that at each jump a new, independent pair of jump length and waiting time are chosen. Globally, anomalous diffusion emerges through scale-free forms of the jump length and/or waiting time distributions by virtue of the generalized central limit theorem. Here we present a modified version of recently proposed correlated CTRW processes, where we incorporate a power-law correlated noise on the level of both jump length and waiting time dynamics. We obtain a very general stochastic model, that encompasses key features of several paradigmatic models of anomalous diffusion: discontinuous, scalefree displacements as in Lévy flights, scale-free waiting times as in subdiffusive CTRWs,

and the long-range temporal correlations of fractional Brownian motion (FBM). We derive the exact solutions for the single-time probability density functions and extract the scaling behaviours. Interestingly, we find that different combinations of the model parameters lead to indistinguishable shapes of the emerging probability density functions and identical scaling laws. Our model will be useful for describing recent experimental single particle tracking data that feature a combination of CTRW and FBM properties.



Leiden University, The Netherlands

# Exact enumeration of Hamiltonian walks on the $4 \times 4 \times 4$ cube and applications to protein folding

#### Raoul D Schram and Helmut Schiessel

2013 J. Phys. A: Math. Theor. 46 485001

Hamiltonian walks on lattices are model systems for compact polymers such as proteins. Here we enumerate exactly the number of Hamiltonian walks on the 4 × 4 × 4 cube and give estimates up to the  $7 \times 7 \times 7$  cube through Monte Carlo methods. We find that the number of configurations grows faster with chain length than previously anticipated. Finally, we discuss uniqueness of ground states in the HP model for protein folding.



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# Chaotic and complex systems



Universität Potsdam. Germany (Section Editor)

Furious development of nonlinear physics in the last decades of the 20th century, due to progress in such fields as chaos, nonlinear pattern formation and turbulence along with the non-trivial role of noise in nonlinear systems, resulted in establishing it, together with statistical physics, as a major methodological interdisciplinary approach to the study of classical dynamical phenomena.

This success story continues in the field of complexity science, fulfilling the expectation of Stephen Hawking, who in 2000 said: "I think the next century will be the century of complexity."

Journal of Physics A offers an optimal platform both for presenting theoretically and mathematically oriented contributions to the basics of nonlinear and complex systems, and for publishing diverse applied studies in this truly interdisciplinary field.

Excellent examples are the recent review article "Networking—a statistical physics perspective" from Yeung and Saad (J. Phys. A: Math. Theor. 46 103001) and the special issue "Lyapunov analysis: from dynamical systems theory to applications". Along with these, this collection also highlights original work on biophysical network models and quantum transport.

#### CHAOTIC AND COMPLEX SYSTEMS



lack Kuipers and Klaus Richter Universität Regensburg, Germany

### Transport moments and Andreev billiards with tunnel barriers

#### **Jack Kuipers and Klaus Richter**

2013 J. Phys. A: Math. Theor. 46 055101

Open chaotic systems are expected to possess universal transport statistics and recently there have been many advances in understanding and obtaining expressions for their transport moments. However, when tunnel barriers are added, which represents the situation in more general experimental physical systems, much less is known about the behaviour of the moments. By incorporating tunnel barriers in the recursive semiclassical diagrammatic approach, we obtain the moment generating function of the transmission eigenvalues at leading and subleading orders. For reflection quantities, quantum mechanical tunnelling phases play an essential role and we introduce new structures to deal with them. This allows us to obtain the moment generating function of the reflection eigenvalues and the Wigner delay times at a leading order. Our semiclassical results are in complementary regimes to the leading order results derived from the random matrix theory expanding the range of theoretically known moments. As a further application, we derive to the leading order the density of states of Andreev billiards coupled to a superconductor through tunnel barriers.



Insight article

iopscience.jop.org/1751-8121/labtalk-article/52776

# On the relation between Lyapunov exponents and exponential decay of correlations

#### Julia Slipantschuk, Oscar F Bandtlow and Wolfram Just

2013 J. Phys. A: Math. Theor. 46 075101

Chaotic dynamics with sensitive dependence on initial conditions may result in exponential decay of correlation functions. We show that for one-dimensional interval maps the corresponding quantities, that is, Lyapunov exponents and exponential decay rates, are related. More specifically, for piecewise linear expanding Markov maps observed via piecewise analytic functions, we show that the decay rate is bounded above by twice the Lyapunov exponent, that is, we establish lower bounds for the subleading eigenvalue of the corresponding Perron-Frobenius operator. In addition, we comment on similar relations for general piecewise smooth expanding maps.

#### **CHAOTIC AND COMPLEX SYSTEMS**



Universidade Federal de São Carlos, Brazil

# Combinatorial problems in the semiclassical approach to quantum chaotic transport

#### **Marcel Novaes**

2013 J. Phys. A: Math. Theor. 46 095101

A semiclassical approach to the calculation of transport moments Mm = Tr[(t+t)m], where t is the transmission matrix, was developed by this author (2012 Europhys. Lett. 98 20006) for chaotic cavities with two leads and broken time-reversal symmetry. The result is an expression for Mm as a perturbation series in 1/N, where N is the total number of open channels, which is in agreement with random matrix theory predictions. The coefficients in this series were related to two open combinatorial problems. Here we expand on this work, including the solution to one of the combinatorial problems. As a byproduct, we also present a conjecture relating two kinds of factorizations of permutations.

### Immune networks: multitasking capabilities near saturation

#### E Agliari, A Annibale, A Barra, A C C Coolen and D Tantari

2013 J. Phys. A: Math. Theor. 46 415003

Pattern-diluted associative networks were recently introduced as models for the immune system, with nodes representing T-lymphocytes and stored patterns representing signalling protocols between T- and B-lymphocytes. It was shown earlier that in the regime of extreme pattern dilution, a system with NTT-lymphocytes can manage a number  $N_p = \mathcal{O}(N_\tau^\delta)$  of B-lymphocytes simultaneously, with  $\delta < 1$ . Here we study this model in the extensive load regime NB =  $\alpha$ NT, with a high degree of pattern dilution, in agreement with immunological findings. We use graph theory and statistical mechanical analysis based on replica methods to show that in the finite-connectivity regime, where each T-lymphocyte interacts with a finite number of B-lymphocytes as NT  $\rightarrow \infty$ , the T-lymphocytes can coordinate effective immune responses to an extensive number of distinct antigen invasions in parallel. As  $\alpha$  increases, the system eventually undergoes a second order transition to a phase with clonal cross-talk interference, where the system's performance degrades gracefully. Mathematically, the model is equivalent to a spin system on a finitely connected graph with many short loops, so one would expect the available analytical methods, which all assume locally tree-like graphs, to fail. Yet it turns out to be solvable. Our results are supported by numerical simulations.

# Mathematical physics



Atsuo Kuniha University of Tokyo. (Section Editor)

In this section, we welcome papers in which the ideas, techniques and applications flow between theoretical physics and mathematics in either direction. We had the pleasure of publishing excellent papers in 2013 with some of the most interesting contributions listed in this Highlights collection. Here is a brief summary from them.

Discrete holomorphic observables are correlation functions in two-dimensional lattice statistical models, which satisfy a discrete analogue of the Cauchy-Riemann relations. They have found numerous applications in recent studies on scaling properties of Ising interfaces, bulk and boundary connectivity constants for self-avoiding walks and so forth. Ikhlef et al. (J. Phys. A: Math. Theor. 46 265205) investigated the Temperley-Lieb loop models and clarified the origin of the discrete holomorphic observables in terms of non-local currents in the relevant quantum group.

Random matrix is the approach that leads to the most universal predictions on the statistical distributions of physically interesting quantities in many complex systems. Akemann et al. (J. Phys. A: Math. Theor. 46 275205) introduced an M-product generalization of the classical Wishart-Laguerre random matrices, which has applications for example to telecommunication of multi-layer multiple input/output channels. The joint probability distribution and correlation functions of the eigenvalues are obtained explicitly in terms of the kernels of biorthogonal polynomials constructed by the hypergeometric and Meijer G-functions.

Tetrahedron equation is a key to the quantum integrability in three dimensions. Its remarkably rich content is still being explored even more than 30 years after the proposal by A B Zamolodchikov in 1980. Bazhanov et al. (J. Phys. A: Math. Theor. 46 465206) constructed a three-dimensional lattice model having positive Boltzmann weights and commuting layer-to-layer transfer matrices from a q-oscillator solution of the tetrahedron equation. Kuniba and Okado (J. Phys. A: Math. Theor. 46 485203) showed that the same solution yields the new quantum R-matrices for the q-oscillator representations of all the rank-one quantum affine algebras by a reduction to two-dimensional systems.



Willard Miller Jr University of Minnesota, USA

# Extended Kepler-Coulomb quantum superintegrable systems in three dimensions

#### E G Kalnins, J M Kress and W Miller Jr

2013 J. Phys. A: Math. Theor. 46 085206

The quantum Kepler-Coulomb system in three dimensions is well known to be second order superintegrable, with a symmetry algebra that closes polynomially under commutators. This polynomial closure is also typical for second order superintegrable systems in 2D and for second order systems in 3D with nondegenerate (four-parameter) potentials. However, the degenerate three-parameter potential for the 3D Kepler-Coulomb system (also second order superintegrable) is an exception, as its symmetry algebra does not close polynomially. The 3D four-parameter potential for the extended Kepler-Coulomb system is not even second order superintegrable, but Verrier and Evans (2008 J. Math. Phys. 49 022902) showed it was fourth order superintegrable, and Tanoudis and Daskaloyannis (2011 arXiv:11020397v1) showed that, if a second fourth order symmetry is added to the generators, the symmetry algebra closes polynomially. Here, based on the Tremblay, Turbiner and Winternitz construction, we consider an infinite class of quantum extended Kepler-Coulomb three- and four-parameter systems indexed by a pair of rational numbers (k1, k2) and reducing to the usual systems when k1 = k2 = 1. We show these systems to be superintegrable of arbitrarily high order and determine the structure of their symmetry algebras. We demonstrate that the symmetry algebras close algebraically; only for systems admitting extra discrete symmetries is polynomial closure achieved. Underlying the structure theory is the existence of raising and lowering operators, not themselves symmetry operators or even defined independent of basis, that can be employed to construct the symmetry operators and their structure relations.

# Two-point correlation function for Dirichlet L-functions

#### E Bogomolny and J P Keating

2013 J. Phys. A: Math. Theor. 46 095202

The two-point correlation function for the zeros of Dirichlet L-functions at a height E on the critical line is calculated heuristically using a generalization of the Hardy-Littlewood conjecture for pairs of primes in arithmetic progression. The result matches the conjectured random-matrix form in the limit as  $E \rightarrow \infty$  and, importantly, includes finite-E corrections. These finite-E corrections differ from those in the case of the Riemann zetafunction, obtained in Bogomolny and Keating (1996 Phys. Rev. Lett. 77 1472), by certain finite products of primes which divide the modulus of the primitive character used to construct the L-function in question.



**Reinout W Quispel** La Trobe University, Australia

# Discrete Painlevé equations and their Lax pairs as reductions of integrable lattice equations

#### C M Ormerod, Peter H van der Kamp and G R W Quispel

2013 J. Phys. A: Math. Theor. 46 095204

We describe a method to obtain Lax pairs for periodic reductions of a rather general class of integrable non-autonomous lattice equations. The method is applied to obtain reductions of the non-autonomous discrete Korteweg-de Vries equation and nonautonomous discrete Schwarzian Korteweg-de Vries equation, which yield a discrete analogue of the fourth Painlevé equation, a q-analogue of the sixth Painlevé equation and the q-Painlevé equation with a symmetry group of affine Weyl type E(1)6.



Peter Forrester The University of Melbourne, Australia

# Skew orthogonal polynomials for the real and quaternion real Ginibre ensembles and generalizations

#### Peter I Forrester

2013 J. Phys. A: Math. Theor. 46 245203

There are some distinguished ensembles of non-Hermitian random matrices for which the joint probability density function can be written down explicitly, are unchanged by rotations, and furthermore which have the property that the eigenvalues form a Pfaffian point process. For these ensembles, in which the elements of the matrices are either real, or real guaternion, the kernel of the Pfaffian is completely determined by certain skew orthogonal polynomials, which permit an expression in terms of averages over the characteristic polynomial, and the characteristic polynomial multiplied by the trace. We use Schur polynomial theory, knowledge of the value of a Schur polynomial averaged against real, and real quaternion Gaussian matrices, and the Selberg integral to evaluate these averages.



Yoichi Nakata The University of Tokyo, Japan

# Solutions to the ultradiscrete KdV equation expressed as the maximum of a quadratic function

#### Yoichi Nakata

2013 J. Phys. A: Math. Theor. 46 265203

We propose the functions defined by the maximum of a discrete quadratic form and satisfying the ultradiscrete KdV equation. These functions include not only soliton solutions but also pseudo-periodic solutions. In the proof, we employ some facts of discrete convex analysis.



Paul 7inn-Justin Université Pierre et Marie Curie, France

### Discrete holomorphicity and quantized affine algebras

#### Y Ikhlef, R Weston, M Wheeler and P Zinn-Justin

2013 J. Phys. A: Math. Theor. 46 265205

We consider non-local currents in the context of quantized affine algebras, following the construction introduced by Bernard and Felder (1991 Nucl. Phys. B 365 98-120). In the case of  $U_2(A_1^{(1)})$  and  $U_2(A_2^{(2)})$ , these currents can be identified with configurations in the 6-vertex and Izergin-Korepin 19-vertex models. Mapping these to their corresponding Temperley-Lieb loop models, we directly identify non-local currents with discretely holomorphic loop observables. In particular, we show that the bulk discrete holomorphicity relation and its recently derived boundary analogue are equivalent to conservation laws for non-local currents.



**Mario Kieburg Bielefeld University** and SUNY, USA



University of Helsinki. Finland and Aalto University, Finland

# Singular value correlation functions for products of Wishart random matrices

#### Gernot Akemann, Mario Kieburg and Lu Wei

2013 J. Phys. A: Math. Theor. 46 275205

We consider the product of M quadratic random matrices with complex elements and no further symmetry, where all matrix elements of each factor have a Gaussian distribution. This generalizes the classical Wishart–Laguerre Gaussian unitary ensemble with M=1. In this paper, we first compute the joint probability distribution for the singular values of the product matrix when the matrix size N and the number M are fixed but arbitrary. This leads to a determinantal point process which can be realized in two different ways. First, it can be written as a one-matrix singular value model with a non-standard Jacobian, or second, for  $M \ge 2$ , as a two-matrix singular value model with a set of auxiliary singular values and a weight proportional to the Meijer G-function. For both formulations, we determine all singular value correlation functions in terms of the kernels of biorthogonal polynomials which we explicitly construct. They are given in terms of the hypergeometric and Meijer G-functions, generalizing the Laguerre polynomials for M = 1. Our investigation was motivated from applications in telecommunication of multi-layered scattering multipleinput and multiple-output channels. We present the ergodic mutual information for finite-N for such a channel model with M-1 layers of scatterers as an example.



Vladimir Mangazeev Australian National University, Australia

# An integrable 3D lattice model with positive Boltzmann weights

#### Vladimir V Mangazeev, Vladimir V Bazhanov and Sergey M Sergeev

2013 J. Phys. A: Math. Theor. 46 465206

In this paper we construct a three-dimensional (3D) solvable lattice model with nonnegative Boltzmann weights. The spin variables in the model are assigned to edges of the 3D cubic lattice and run over an infinite number of discrete states. The Boltzmann weights satisfy the tetrahedron equation, which is a 3D generalization of the Yang-Baxter equation. The weights depend on a free parameter 0 < q < 1 and three continuous field variables. The layer-to-layer transfer matrices of the model form a two-parameter commutative family. This is the first example of a non-trivial solvable 3D lattice model with non-negative Boltzmann weights.



**Emil Prodan** Yeshiva University, IISA

# The non-commutative nth-Chern number ( $n \ge 1$ )

#### **Emil Prodan, Bryan Leung and Jean Bellissard**

2013 J. Phys. A: Math. Theor. 46 485202

The theory of the higher Chern numbers in the presence of strong disorder is developed. Sharp quantization and homotopy invariance conditions are provided. The relevance of the result to the field of strongly disordered topological insulators is discussed.



ionscience.iop.org/1751-8121/labtalk-article/56267



Atsuo Kuniba University of Tokyo,

Tetrahedron equation and quantum R matrices for infinitedimensional modules of  $U_a(A_1^{(1)})$  and  $U_a(A_2^{(2)})$ 

#### Atsuo Kuniba and Masato Okado

2013 J. Phys. A: Math. Theor. 46 485203

From the *q*-oscillator solution to the tetrahedron equation associated with a quantized coordinate ring, we construct solutions to the Yang-Baxter equation by applying a reduction procedure formulated earlier by Sergeev and the first author. The results are identified with the quantum R matrices for the infinite-dimensional modules of  $Uq(A_{\star}^{(1)})$ and  $Uq(A_0^{(2)})$  corresponding to an affinization of Verma modules of their subalgebras isomorphic to  $Uq(sl_2)$  and  $Uq^4(sl_2)$ .

# Quantum mechanics and quantum information theory



University of Siegen. Germany (Section Editor)

The quantum mechanics and quantum information section of the journal publishes a broad spectrum of papers, from research that considers the foundational and conceptual ideas underlying quantum mechanics to articles that discuss aspects of quantum information as diverse as entanglement measures, quantum communication and cryptography.

This collection features papers from across this spectrum: these include Sarbicki and Chruscinski (J. Phys. A: Math. Theor. 46 015306) who look at the properties of indecomposable maps, which can be used to explore entanglement, and Tomamichel and Hänggi (J. Phys. A: Math. Theor. 46 055301) who consider an entropic uncertainty relation and its applications to quantum cryptography. Deffner and Lutz (J. Phys. A: Math. Theor. 46 335302) have studied the quantum speed limit time finding generalisations of the energy-time uncertainty relation for driven quantum systems, and Niekamp et al. (J. Phys. A: Math. Theor. 46 125301) characterize the complexity of multiparticle quantum states using concepts from information geometry.

This collection also includes high-quality research on fundamental and general aspects of quantum mechanics. For example, Braak (J. Phys. A: Math. Theor. 46 175301) considers the spectrum of the well known quantum Rabi model using continued fractions, while Zhong et al. (J. Phys. A: Math. Theor. 46 415302) develop a method to find analytical solutions for the eigenstates of the same model. Finally, Hollowood (J. Phys. A: Math. Theor. 46 325302) discusses the Copenhagen interpretation of quantum mechanics as an emergent phenomenon.

We are pleased to have the opportunity to highlight some of the excellent work published in this broad field in 2013 and we hope that you enjoy reading these papers. We look forward to publishing more high-quality work in the section throughout 2014.



Dariusz Chruściński Nicolaus Copernicus University, Poland

# A class of exposed indecomposable positive maps

#### Gniewomir Sarbicki and Dariusz Chruściński

2013 J. Phys. A: Math. Theor. 46 015306

Exposed positive maps in matrix algebras define a dense subset of extremal maps. We provide a class of indecomposable positive maps in the algebra of 2n × 2n complex matrices with  $n \ge 2$ . It is shown that these maps are exposed and hence define the strongest tool in entanglement theory to discriminate between separable and entangled states.



National University of Singapore and ETH Zurich, Switzerland

### The link between entropic uncertainty and nonlocality

#### Marco Tomamichel and Esther Hänggi

2013 J. Phys. A: Math. Theor. 46 015306

Two of the most intriguing features of quantum physics are the uncertainty principle and the occurrence of nonlocal correlations. The uncertainty principle states that there exist pairs of incompatible measurements on quantum systems such that their outcomes cannot both be predicted. On the other hand, nonlocal correlations of measurement outcomes at different locations cannot be explained by classical physics, but appear in the presence of entanglement. Here, we show that these two fundamental quantum effects are quantitatively related. Namely, we provide an entropic uncertainty relation for the outcomes of two binary measurements, where the lower bound on the uncertainty is quantified in terms of the maximum Clauser-Horne-Shimony-Holt value that can be achieved with these measurements. We discuss applications of this uncertainty relation in quantum cryptography, in particular, to certify quantum sources using untrusted devices.



Polish Academy of Sciences, Poland

# When is a pure state of three qubits determined by its single-particle reduced density matrices?

#### A Sawicki, M Walter and M Kuś

2013 J. Phys. A: Math. Theor. 46 055304

Using techniques from symplectic geometry, we prove that a pure state of three qubits is up to local unitaries uniquely determined by its one-particle reduced density matrices exactly when their ordered spectra belong to the boundary of the so-called Kirwan polytope. Otherwise, the states with given reduced density matrices are parameterized, up to local unitary equivalence, by two real variables. Given inevitable experimental imprecision, this means that already for three gubits a pure quantum state can never be reconstructed from single-particle tomography. We moreover show that the knowledge of the reduced density matrices is always sufficient if one is given the additional promise that the quantum state is not convertible to the Greenberger-Horne-Zeilinger state by stochastic local operations and classical communication, and discuss generalizations of our results to an arbitrary number of gubits.



Otfried Gühne Universität Siegen, Germany

# Computing complexity measures for quantum states based on exponential families

Sönke Niekamp, Tobias Galla, Matthias Kleinmann and Otfried Gühne

2013 J. Phys. A: Math. Theor. 46 125301

Given a multiparticle quantum state, one may ask whether it can be represented as a thermal state of some Hamiltonian with k-particle interactions only. The distance from the exponential family defined by these thermal states can be considered as a measure of complexity of a given state. We investigate the resulting optimization problem and show how symmetries can be exploited to simplify the task of finding the nearest thermal state in a given exponential family. We also present an algorithm for the computation of the complexity measure and consider specific examples to demonstrate its applicability.



University of Augsburg, Germany

#### Continued fractions and the Rabi model

#### Daniel Braak

2013 J. Phys. A: Math. Theor. 46 175301

Techniques based on continued fractions to numerically compute the spectrum of the quantum Rabi model are reviewed. They are of two essentially different types. In the first case, the spectral condition is implemented using a representation in the infinite-dimensional Bargmann space of analytic functions. This approach is shown to approximate the correct spectrum of the full model if the continued fraction is truncated at sufficiently high order. In the second case, one considers the limit of a sequence of models defined in finite-dimensional state spaces. In contrast to the first, the second approach is ambiguous and can be justified only through recourse to the analyticity argument from the first method.



CEA/DAM/DIF, France

# Convexity and the quantum many-body problem

#### P Chau Huu-Tai and P Van Isacker

2013 J. Phys. A: Math. Theor. 46 205302

We recall some properties of convex functions and, in particular, of the sum of the largest eigenvalues of a Hermitian matrix. From these properties a new estimate of an arbitrary eigenvalue of a sum of Hermitian matrices is derived, which in turn is used to compute an approximate associated spectral projector. These estimates are applied for the first time to explain the generic spectral features of quantum systems. As an application of the formalism, we explain the preponderance of certain ground-state angular momenta as observed in the vibron model with random interactions. We show that the evolution of eigenstates can be predicted from the knowledge of a limited number of spectra and investigate the effect of a three-body interaction in the vibron model on eigenenergies and eigenvectors.



Timothy J Hollowood Swansea University,

# The Copenhagen interpretation as an emergent phenomenon

#### **Timothy J Hollowood**

2013 J. Phys. A: Math. Theor. 46 325302

The Copenhagen interpretation has been remarkably successful but seems at odds with the underlying linearity of quantum mechanics. We show how it can emerge in a simple way from the underlying microscopic quantum world governed by Schrödinger's equation without the need for observers or their brains. In order to achieve this, we assemble pieces of various pre-existing ideas. Firstly, we adopt a relational approach and use the eigenvectors of the reduced density matrix of a quantum sub-system, or equivalently the Schmidt decomposition, to define the 'internal state' of a sub-system. Previous work has identified serious objections to such an interpretation because it apparently leads to macroscopic superpositions and physically unacceptable instabilities near degeneracies. We show that both these problems are solved if the sub-system consists of a large number of coarse-grained degrees of freedom as one expects in order to make contact with the classical world. We further argue that coarse graining is a necessary ingredient because measuring devices have both finite spatial and temporal resolutions. What results is an interpretation in which both decoherence and coarse graining play key roles and from which the rules of the Copenhagen interpretation are seen to emerge in realistic situations that include the measurement of the position of a particle and a decay process.



University of Augsburg, Germany and University of Maryland, USA

# Energy—time uncertainty relation for driven quantum systems

#### Sebastian Deffner and Eric Lutz

2013 J. Phys. A: Math. Theor. 46 335302

We derive generalizations of the energy-time uncertainty relation for driven quantum systems. Using a geometric approach based on the Bures length between mixed quantum states, we obtain explicit expressions for the quantum speed limit time, valid for arbitrary initial and final quantum states and arbitrary unitary driving protocols. Our results establish the fundamental limit on the rate of evolution of closed quantum systems.



**Chaohong Lee** Sun Yat-Sen University, People's Republic of China

# Analytical eigenstates for the quantum Rabi model

#### Honghua Zhong, Qiongtao Xie, Murray T Batchelor and Chaohong Lee

2013 J. Phys. A: Math. Theor. 46 415302

We develop a method to find analytical solutions for the eigenstates of the quantum Rabi model. These include symmetric, anti-symmetric and asymmetric analytic solutions given in terms of the confluent Heun functions. Both regular and exceptional solutions are given in a unified form. In addition, the analytic conditions for determining the energy spectrum are obtained. Our results show that conditions proposed by Braak (2011 Phys. Rev. Lett. **107** 100401) are a type of sufficiency condition for determining the regular solutions. The well-known Judd isolated exact solutions appear naturally as truncations of the confluent Heun functions.



**Jussi Schultz** Politecnico di Milano, Italy

## Informationally complete sets of Gaussian measurements

#### Jukka Kiukas and Jussi Schultz

2013 J. Phys. A: Math. Theor. 46 485303

We prove the necessary and sufficient conditions for the informational completeness of an arbitrary set of Gaussian observables on continuous variable systems with a finite number of degrees of freedom. In particular, we show that an informationally complete set either contains a single informationally complete observable, or includes infinitely many observables. We show that for a single informationally complete observable, the minimal outcome space is the phase space, and the corresponding probability distribution can always be obtained from the quantum optical Q-function by linear postprocessing and Gaussian convolution, in a suitable symplectic coordinatization of the phase space. In the case of projection valued Gaussian observables, e.g., generalized field quadratures, we show that an informationally complete set of observables is necessarily infinite. Finally, we generalize the treatment to the case where the measurement coupling is given by a general linear bosonic channel, and characterize informational completeness for an arbitrary set of the associated observables.

# Field theory and string theory



**Arkady Tseytlin** Imperial College London, UK (Section Editor)

The field theory and string theory section publishes high-quality and significant new results in areas including general methods of quantum field theory, supersymmetric field theories in various dimensions, conformal field theories, integrable theories, string theory and its applications.

In addition to the traditionally strong area of gauge-string duality and integrability, the section attracts papers on the currently active subjects of scattering amplitudes, higher spins theories and their conformal theory connections, and on exact methods in field theory, like localization. Each of these areas has been or will be featured in a special issue of the journal.

### The geometry of the limit of N = 2 minimal models

#### Stefan Fredenhagen and Cosimo Restuccia

2013 J. Phys. A: Math. Theor. 46 045402

We consider the limit of two-dimensional N = (2, 2) superconformal minimal models when the central charge approaches c = 3. Starting from a geometric description as nonlinear sigma models, we show that one can obtain two different limit theories. One is the free theory of two bosons and two fermions, the other one is a continuous orbifold thereof. We substantiate this claim by detailed conformal field theory computations.

# The 1/2 BPS 't Hooft loops in $\mathcal{N}$ = 4 SYM as instantons in 2D Yang-Mills

#### **Simone Giombi and Vasily Pestun**

2013 J. Phys. A: Math. Theor. 46 095402

We extend the recent conjecture on the relation between a certain 1/8 BPS subsector of 4D  $\mathcal{N}$ =4 SYM on S2 and 2D Yang–Mills theory by turning on circular 1/2 BPS 't Hooft operators linked with S2. We show that localization predicts that these 't Hooft operators and their correlation functions with Wilson operators on S2 are captured by instanton contributions to the partition function of the 2D Yang-Mills theory. Based on this prediction, we compute explicitly correlation functions involving the 't Hooft operator, and observe precise agreement with S-duality predictions.

#### FIELD THEORY AND STRING THEORY



Valentina Forini Universitat de Barcelona, Spain and Humboldt-Universität zu Berlin, Germany

# The generalized cusp in $AdS_4 \times \mathbb{C}P^3$ and more one-loop results from semiclassical strings

#### V Forini, V Giangreco M Puletti and O Ohlsson Sax

2013 J. Phys. A: Math. Theor. 46 115402

We evaluate the exact one-loop partition function for fundamental strings whose world surface ends on a cusp at the boundary of AdS4 and has a 'jump' in . This allows us to extract the stringy prediction for the ABJM-generalized cusp anomalous dimension  $\Gamma$ ABJMcusp(phgr,  $\theta$ ) up to NLO in sigma-model perturbation theory. With a similar analysis, we present the exact partition functions for folded closed string solutions moving in the AdS3 parts of and AdS3 × S3 × S3 × S1 backgrounds. Results are obtained applying to the string solutions relevant for the AdS4/CFT3 and AdS3/CFT2 correspondence the tools previously developed for their AdS5 × S5 counterparts.



**Arkady A Tseytlin** Imperial College London, UK

# On (non)integrability of classical strings in p-brane backgrounds

#### A Stepanchuk and A A Tsevtlin

2013 J. Phys. A: Math. Theor. 46 125401

We investigate the question of possible integrability of classical string motion in curved p-brane backgrounds. For example, the D3-brane metric interpolates between the flat and the AdS5 × S5 regions in which string propagation is integrable. We find that while the point-like string (geodesic) equations are integrable, the equations describing an extended string in the complete D3-brane geometry are not. The same conclusion is reached for similar brane intersection backgrounds interpolating between flat space and AdSk × Sk. We consider, in particular, the case of the NS 5-brane—fundamental string background. To demonstrate non-integrability we make a special 'pulsating string' ansatz for which the string equations reduce to an effective one-dimensional system. Expanding near this simple solution leads to a linear differential equation for small fluctuations that cannot be solved in quadratures, implying non-integrability of the original set of string equations.

#### FIELD THEORY AND STRING THEORY



**Martin Speight** University of Leeds,

# Isospinning hopfions

#### Derek Harland, Juha Jävkkä, Yakov Shnir and Martin Speight

2013 J. Phys. A: Math. Theor. 46 225402

The problem of constructing internally rotating solitons of fixed angular frequency  $\omega$  in the Faddeev-Skyrme model is reformulated as a variational problem for an energy-like functional, called pseudoenergy, which depends parametrically on  $\omega$ . This problem is solved numerically using a gradient descent method, without imposing any spatial symmetries on the solitons, and the dependence of the solitons' energy on  $\omega$ , and on their conserved total isospin J. studied. It is found that, generically, the shape of a soliton is independent of  $\omega$ , and that its size grows monotonically with  $\omega$ . A simple elastic rod model of time-dependent hopfions is developed which, despite having only one free parameter, accounts well for most of the numerical results.



Uppsala University, Sweden

# 5D super Yang-Mills theory and the correspondence to AdS<sub>2</sub>/CFT<sub>c</sub>

#### Joseph A Minahan, Anton Nedelin and Maxim Zabzine

2013 J. Phys. A: Math. Theor. 46 355401

We study the relation between 5D super Yang-Mills theory and the holographic description of 6D (2, 0) superconformal theory. We start by clarifying some issues related to the localization of  $\mathcal{N}$  = 1 SYM with matter on S5. We concentrate on the case of a single adjoint hypermultiplet with a mass term and argue that the theory has a symmetry enlargement at mass M = 1/(2r), where r is the S5 radius. However, in order to have a well-defined localization locus it is necessary to rotate M onto the imaginary axis, breaking the enlarged symmetry, Based on our prescription, the imaginary mass values are physical and we show how the localized path integral is consistent with earlier results for 5D SYM in flat space. We then compute the free energy and the expectation value for a circular Wilson loop in the large N limit. The Wilson loop calculation shows a mass dependent constant rescaling between weak and strong coupling. The Wilson loop continued back to to the enlarged symmetry point is consistent with a supergravity computation for an M2 brane using the standard identification of the compactification radius and the 5D coupling. If we continue back to the physical regime and use this value of the mass to determine the compactification radius, then we find agreement between the SYM free energy and the corresponding supergravity calculation. We also verify numerically some of our analytic approximations.

#### FIELD THEORY AND STRING THEORY



John Cardy University of Oxford.

# Some results on the mutual information of disjoint regions in higher dimensions

#### **John Cardy**

2013 J. Phys. A: Math. Theor. 46 285402

We consider the mutual Rényi information  $I^{(n)}$  (A,B)  $\equiv S_A^{(n)} + S_B^{(n)} - S_{AUB}^{(n)}$  of disjoint compact spatial regions A and B in the ground state of a d + 1-dimensional conformal field theory (CFT), in the limit when the separation r between A and B is much greater than their sizes RA, B. We show that in general  $I^{(n)}$  (A,B)  $\sim C_A^{(n)} C_B^{(n)} R_A R_B / r^2)^{\alpha}$ , where  $\alpha$  is the smallest sum of the scaling dimensions of operators whose product has the quantum numbers of the vacuum, and the constants  $C_{\rm AB}^{\rm (n)}$  depend only on the shape of the regions and universal data of the CFT. For a free massless scalar field, where  $\alpha = d - 1$ , we show that  $C^{(2)}_{_{A}}R^{_{A}}_{_{A}}$  is proportional to the capacitance of a thin conducting slab in the shape of A in d + 1-dimensional electrostatics, and give explicit formulae for this when A is the interior of a sphere Sd - 1 or an ellipsoid. For spherical regions in d = 2 and 3 we obtain explicit results for C(n) for all n and hence for the leading term in the mutual information by taking n → 1. We also compute a universal logarithmic correction to the area law for the Rényi entropies of a single spherical region for a scalar field theory with a small mass.



**David Foster** University of Kent, UK



University of Kent, UK

# Negative baryon density and the folding structure of the B = 3 skyrmion

#### D Foster and S Krusch

2013 J. Phys. A: Math. Theor. 46 265401

The Skyrme model is a nonlinear field theory whose solitonic solutions, once quantized, describe atomic nuclei. The classical static soliton solutions, so-called skyrmions, have interesting symmetries and can only be calculated numerically. Mathematically, these skyrmions can be viewed as maps between two three-manifolds and, as such, their stable singularities can only be folds, cusps and swallowtails. Physically, the occurrence of singularities is related to negative baryon density. In this paper, we calculate the charge three skyrmion to a high resolution in order to examine its singularity structure in detail. Thereby, we explore regions of negative baryon density. We also discuss how the negative baryon density depends on the pion mass.



Insight article

iopscience.iop.org/1751-8121/labtalk-article/53922

# Fluid and plasma theory



**Gregory Falkovich** Weizmann Institute of Sciences, Israel (Section Editor)

The fluid and plasma theory section of Journal of Physics A offers a place for researchers working in these areas across a wide range of disciplines to interact with each other, and a place to publish theoretical and mathematical contributions in their diverse fields.

Theory of fluids involves a very wide scope of mathematical subjects and theoretical methods, from topology, algebra and integrability to modern stochastic methods, as illustrated by the excellent papers in this collection.

A new topological analysis sheds new light on transport in magnetically confined turbulence. The analysis is done by separating the structures into radial layers and studying each layer separately. This allows for the identification of flow cycles and flow filaments and the determination of the life of the cycles and length of the filaments (A topological analysis of plasma flow structures, Benjamin A Carreras, Irene Llerena Rodríguez and Luis García 2013 J. Phys. A: Math. Theor. 46 375501).

Application of field-theoretical methods to turbulence statistics is advanced by identifying instantons in direct numerical simulations of the stochastically driven Burgers equation (Instanton filtering for the stochastic Burgers equation, Tobias Grafke, Rainer Grauer and Tobias Schäfer 2013 J. Phys. A: Math. Theor. 46 062002).

Classical fluid dynamics is an ever-green field, as shown in the paper devoted to the consistent derivation of the equation for long waves on the surface of a shallow layer of an ideal fluid moving under the influence of gravity as well as surface tension (Ordering of two small parameters in the shallow water wave problem. Georgy I Burde and Artur Sergyeyev 2013 J. Phys. A: Math. Theor. 46 075501).

Algebraic methods found a new application in identifying discrete clusters of quasiresonant triads arising from a Hamiltonian three-wave equation (Quadratic invariants for discrete clusters of weakly interacting waves, Katie L Harper, Miguel D Bustamante and Sergey V Nazarenko 2013 J. Phys. A: Math. Theor. 46 245501).

#### **FLUID AND PLASMA THEORY**



**Tobias Schäfer** Ruhr-Universität Bochum, Germany

#### Instanton filtering for the stochastic Burgers equation

#### **Tobias Grafke, Rainer Grauer and Tobias Schäfer**

2013 J. Phys. A: Math. Theor. 46 062002

We address the question of whether one can identify instantons in direct numerical simulations of the stochastically driven Burgers equation. For this purpose, we first solve the instanton equations using the Chernykh–Stepanov method (2001 Phys. Rev. E 64 026306). These results are then compared to direct numerical simulations by introducing a filtering technique to extract prescribed rare events from massive data sets of realizations. Using this approach we can extract the entire time history of the instanton evolution, which allows us to identify the different phases predicted by the direct method of Chernykh and Stepanov with remarkable agreement.



Katie L Harper University of Warwick.

#### Ouadratic invariants for discrete clusters of weakly interacting waves

#### Katie L Harper, Miguel D Bustamante and Sergey V Nazarenko

2013 J. Phys. A: Math. Theor. 46 245501

We consider discrete clusters of quasi-resonant triads arising from a Hamiltonian threewave equation. A cluster consists of N modes forming a total of M connected triads. We investigate the problem of constructing a functionally independent set of quadratic constants of motion. We show that this problem is equivalent to an underlying basic linear problem, consisting of finding the null space of a rectangular M  $\times$  N matrix A with entries 1, -1 and 0. In particular, we prove that the number of independent quadratic invariants is equal to  $J = N - M^* \ge N - M$ , where  $M^*$  is the number of linearly independent rows in  $\mathbb{A}$ . Thus, the problem of finding all independent quadratic invariants is reduced to a linear algebra problem in the Hamiltonian case. We establish that the properties of the quadratic invariants (e.g., locality) are related to the topological properties of the clusters (e.g., types of linkage). To do so, we formulate an algorithm for decomposing large clusters into smaller ones and show how various invariants are related to certain parts of a cluster, including the basic structures leading to M\* < M. We illustrate our findings by presenting examples from the Charney-Hasegawa-Mima wave model, and by showing a classification of small (up to three-triad) clusters.

#### **FLUID AND PLASMA THEORY**



Georgy I Burde Ben-Gurion University, Israel

#### Ordering of two small parameters in the shallow water wave problem

#### Georgy I Burde and Artur Sergyeyev

2013 J. Phys. A: Math. Theor. 46 075501

The classical problem of irrotational long waves on the surface of a shallow layer of an ideal fluid moving under the influence of gravity as well as surface tension is considered. A systematic procedure for deriving an equation for surface elevation for a prescribed relation between the orders of the two expansion parameters, the amplitude parameter  $\alpha$ and the long wavelength (or shallowness) parameter  $\beta$ , is developed. Unlike the heuristic approaches found in the literature, when modifications are made in the equation for surface elevation itself, the procedure starts from the consistently truncated asymptotic expansions for unidirectional waves, a counterpart of the Boussinesg system of equations for the surface elevation and the bottom velocity, from which the leading-order and higher order equations for the surface elevation can be obtained by iterations. The relations between the orders of the two small parameters are taken in the form  $\beta = O(\alpha n)$  and  $\alpha = O(\beta m)$  with n and m specified to some important particular cases. The analysis shows, in particular, that some evolution equations, proposed before as model equations in other physical contexts (such as the Gardner equation, the modified Korteweg-de Vries (KdV) equation and the so-called fifth-order KdV equation), can emerge as the leadingorder equations in the asymptotic expansion for the unidirectional water waves, on equal footing with the KdV equation. The results related to the higher orders of approximation provide a set of consistent higher order model equations for unidirectional water waves which replace the KdV equation with higher order corrections in the case of non-standard ordering when the parameters  $\alpha$  and  $\beta$  are not of the same order of magnitude. The shortcomings of certain models used in the literature become apparent as a result of the subsequent analysis. It is also shown that various model equations obtained by assuming a prescribed relation  $\beta = O(\alpha n)$  between the orders of the two small parameters can be equivalently treated as obtained by applying transformations of variables which scale out the parameter  $\beta$ , in favor of  $\alpha$ . It allows us to consider the nonlinearity-dispersion balance, epitomized by the soliton equations, as existing for any  $\beta$ , provided that  $\alpha \rightarrow 0$ , but leads to a prescription, in asymptotic terms, of the region of time and space where the equations are valid and so the corresponding dynamics are expected to occur.

#### **FLUID AND PLASMA THEORY**



**Benjamin A Carreras** Universidad Carlos III de Madrid, Spain

#### A topological analysis of plasma flow structures

Benjamin A Carreras, Irene Llerena Rodríguez and Luis García

2013 J. Phys. A: Math. Theor. 46 375501

The analysis is done by separating the structures into radial layers and studying each layer separately. This allows for the identification of flow cycles and flow filaments and the determination of the life of the cycles and length of the filaments.



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ABJ triality: from higher spin fields to strings, Chi-Ming Chang, Shiraz Minwalla, Tarun Sharma and Xi Yin 2013 J. Phys. A: Math. Theor. 46 214009

Constraining conformal field theories with a higher spin symmetry, Juan Maldacena and Alexander Zhiboedov 2013 J. Phys. A: Math. Theor. 46 214011



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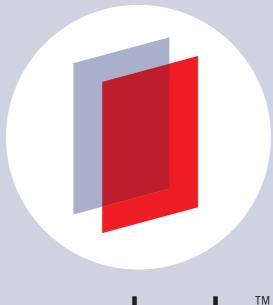
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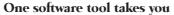
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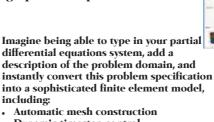


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