Journal of Physics A
Mathematical and Theoretical

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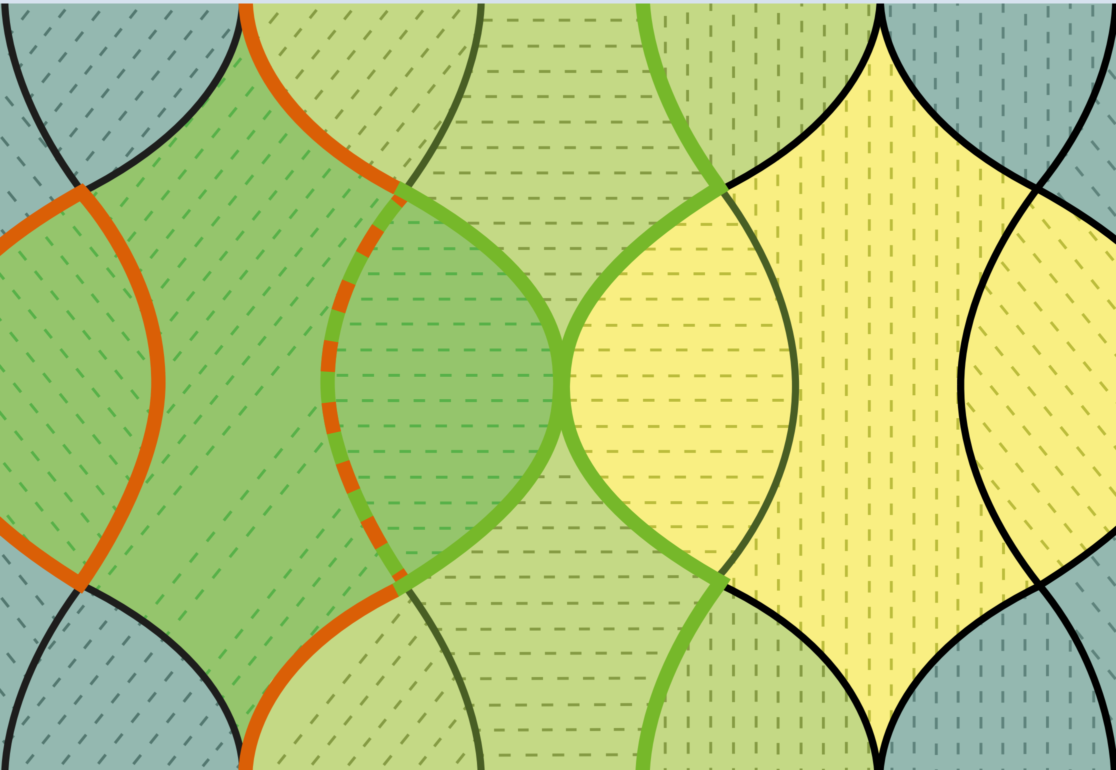
Highlights

A collection of outstanding research published in 2014



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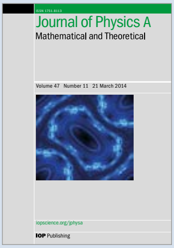

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Welcome

Martin Evans
Editor-in-Chief

Welcome to the *Journal of Physics A: Mathematical and Theoretical* (JPhysA) Highlights collection. This collection showcases some of the most highly rated articles published in the journal over the course of 2014.

This selection of articles displays the broad scope of the journal and demonstrates how it is a meeting place for researchers to share mathematically rich work across different disciplines.

Some of our most significant articles have been published as Fast Track Communications (FTCs). We are pleased to highlight the newly relaunched Fast Track Communications Section for 2015. Many authors in theoretical and mathematical physics communities have expressed a desire for a prestigious, dedicated venue for short, pioneering contributions, not restricted by 'general interest criteria'. We welcome contributions reporting new and timely developments in mathematically rich theoretical physics. Published FTCs will be distinguished from regular articles by "FT" inserted into the article number. We have appointed a dedicated Editorial Panel of highly respected scientists who will assess all FTC submissions. We invite you to submit your work to this section.

Also included in this collection are details of the topical reviews published in 2014. These commissioned topical reviews provide timely overviews of the current state of research in areas of great interest and activity. In 2014, we also published two high-quality special issues on '50 years of Bell's theorem' and 'Cluster algebras in mathematical physics'.

We hope that you will enjoy this celebration of highlighted articles.

From the Publisher

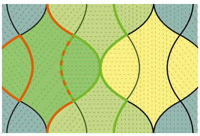
Rebecca Gillan
Publisher

We published some excellent papers in 2014. We'd like to thank all of our authors for choosing to submit their high-quality work to the journal, and thank our referees and board members for providing constructive peer review and maintaining the quality standards of the journal.

We hope that you will find this collection of articles interesting and we invite you to consider JPhysA for your next paper.

We look forward to working with you during 2015.

Contents



Cover image: Two-particle bound states on the rapidity torus
Stijn J van Tongeren 2014 *J. Phys. A: Math. Theor.* **47** 433001

Editorial Board

Fast Track Communications

Rafael I Nepomechie

Boundary energy of the open XXX chain with a non-diagonal boundary term

Rafael I Nepomechie and Chunguang Wang

2014 *J. Phys. A: Math. Theor.* **47** 032001

We analyze the ground state of the open spin-1/2 isotropic quantum spin chain with a non-diagonal boundary term using a recently proposed Bethe ansatz solution. As the coefficient of the non-diagonal boundary term tends to zero, the Bethe roots split evenly into two sets: those that remain finite, and those that become infinite. We argue that the former satisfy conventional Bethe equations, while the latter satisfy a generalization of the Richardson–Gaudin equations. We derive an expression for the leading correction to the boundary energy in terms of the boundary parameters.

Hirohiko Shimada

Phase diagram and strong-coupling fixed point in the disordered O(n) loop model

H Shimada, J L Jacobsen and Y Kamiya

2014 *J. Phys. A: Math. Theor.* **47** 122001

We study the phase diagram and critical properties of the two-dimensional disordered O(n) loop model. The renormalization group (RG) flow is extracted from the landscape of the effective central charge c obtained by the transfer matrix method. We find a line of multicritical fixed points (FPs) at strong randomness for $n > n_c \sim 0.5$. We also find a line of stable random FPs for $n_c < n < 1$, whose c and critical exponents agree well with the $1 - n$ expansion results. The multicritical FP at $n = 1$ has $c = 0.4612(4)$, which suggests that it belongs to the universality class of the Nishimori point in the random-bond Ising model. For $n > 2$, we find another critical line that connects the hard-hexagon FP in the pure model to a finite-randomness zero-temperature FP.

FAST TRACK COMMUNICATIONS

Darren G Crowdy

Vortex patch equilibria of the Euler equation and random normal matrices

Darren G Crowdy

2014 *J. Phys. A: Math. Theor.* **47** 212002

A new analogy is drawn between vortex patch, or V-state, equilibria of the Euler equations for ideal fluids and the planar limit of random normal matrix models. Physically the former is a quite different fluid dynamical problem to Hele-Shaw flow, or Laplacian growth, to which an analogy with matrix models has become well known in recent years. The connection of random matrices with vortex dynamics is made via the so-called modified Schwarz potential. This theoretical link, while interesting in itself, has immediate ramifications for random matrix theory by virtue of a transfer of mathematical technology already well developed for vortex dynamics. Hence for multi-support matrix models in the planar limit we describe a constructive approach to the inverse problem of finding the shapes of the eigenvalue supports for a given potential using automorphic conformal mappings within a Schottky model of the underlying algebraic curves and use of the Schottky–Klein prime function. Two-support eigenvalue distributions in a quartic potential are given as an illustrative example.

Shamik Gupta

Diffusion in periodic, correlated random forcing landscapes

David S Dean, Shamik Gupta, Gleb Oshanin, Alberto Rosso and Grégory Schehr

2014 *J. Phys. A: Math. Theor.* **47** 372001

We study the dynamics of a Brownian particle in a strongly correlated quenched random potential defined as a periodically–extended (with period L) finite trajectory of a fractional Brownian motion with arbitrary Hurst exponent $H \in (0, 1)$. While the periodicity ensures that the ultimate long–time behavior is diffusive, the generalized Sinai potential considered here leads to a strong logarithmic confinement of particle trajectories at intermediate times. These two competing trends lead to dynamical frustration and result in a rich statistical behavior of the diffusion coefficient D_L: although one has the typical value $D_L^{typ} \sim \exp(-\beta L^H)$, we show via an exact analytical approach that the positive moments ($k > 0$) scale like $< D_L^k > \sim \exp[-c'(k\beta L^H)^{1/(1+H)}]$, and the negative ones as $< D_L^{-k} > \sim \exp(\alpha'(k\beta L^H)^2)$, c' and α' being numerical constants and β the inverse temperature. These results demonstrate that D_L is strongly non-self-averaging. We further show that the probability distribution of D_L has a log–normal left tail and a highly singular, one–sided log–stable right tail reminiscent of a Lifshitz singularity.

FAST TRACK COMMUNICATIONS

Dispersionless DKP hierarchy and the elliptic Löwner equation

V Akhmedova and A Zabrodin

2014 *J. Phys. A: Math. Theor.* **47** 392001

We show that the dispersionless DKP hierarchy (the dispersionless limit of the Pfaff lattice) admits a suggestive reformulation through elliptic functions. We also consider one-variable reductions of the dispersionless DKP hierarchy and show that they are described by an elliptic version of the Löwner equation. With a particular choice of the driving function, the latter appears to be closely related to the Painlevé VI equation with a special choice of parameters.

Beatriz Olmos

Effective dynamics of strongly dissipative Rydberg gases

M Marcuzzi, J Schick, B Olmos and I Lesanovsky

2014 *J. Phys. A: Math. Theor.* **47** 482001

We investigate the evolution of interacting Rydberg gases in the limit of strong noise and dissipation. Starting from a description in terms of a Markovian quantum master equation we derive effective equations of motion that govern the dynamics on a ‘coarse-grained’ timescale where fast dissipative degrees of freedom have been adiabatically eliminated. Specifically, we consider two scenarios which are of relevance for current theoretical and experimental studies—Rydberg atoms in a two-level (spin) approximation subject to strong dephasing noise as well as Rydberg atoms under so-called electromagnetically induced transparency (EIT) conditions and fast radiative decay. In the former case we find that the effective dynamics is described by classical rate equations up to second order in an appropriate perturbative expansion. This drastically reduces the computational complexity of numerical simulations in comparison to the full quantum master equation. When accounting for the fourth order correction in this expansion, however, we find that the resulting equation breaks the preservation of positivity and thus cannot be interpreted as a proper classical master rate equation. In the EIT system we find that the expansion up to second order retains information not only on the ‘classical’ observables, but also on some quantum coherences. Nevertheless, this perturbative treatment still achieves a non-trivial reduction of complexity with respect to the original problem.

FAST TRACK COMMUNICATIONS

Matteo Beccaria

Vectorial AdS_5/CFT_4 duality for spin-one boundary theory

Matteo Beccaria and Arkady A Tseytlin

2014 *J. Phys. A: Math. Theor.* **47** 492001

We consider an example of vectorial AdS_5/CFT_4 duality when the boundary theory is described by free N complex or real Maxwell fields. It is dual to a particular ('type C') higher spin theory in AdS_5 containing fields in special mixed-symmetry representations. We extend the study of this theory by Beccaria and Tseytlin (2014 Higher spins in AdS_5 at one loop: vacuum energy, boundary conformal anomalies and AdS/CFT arXiv:1410.3273) by deriving the expression for the large N limit of the corresponding singlet-sector partition function on $S^1 \times S^3$. We find that in both complex $U(N)$ and real $O(N)$ invariant cases the form of the one-particle partition function is as required by the AdS/CFT duality. We also discuss the matching of the Casimir energy on S^3 by assuming an integer shift in the bulk theory coupling.

Ralf Metzler

Localisation and universal fluctuations in ultraslow diffusion processes

Aljaž Godec, Aleksei V Chechkin, Eli Barkai, Holger Kantz and Ralf Metzler

2014 *J. Phys. A: Math. Theor.* **47** 492002

We study ultraslow diffusion processes with logarithmic mean squared displacement (MSD) $\langle x^2(t)\rangle \simeq \log^\gamma t$. Comparison of annealed (renewal) continuous time random walks (CTRWs) with logarithmic waiting time distribution $\psi(\tau) \simeq 1/(\tau \log^{1+\gamma}\tau)$ and Sinai diffusion in quenched random landscapes reveals striking similarities, despite the great differences in their physical nature. In particular, they exhibit a weakly non-ergodic disparity of the time-averaged and ensemble-averaged MSDs. Remarkably, for the CTRW we observe that the fluctuations of time averages become universal, with an exponential suppression of mobile trajectories. We discuss the fundamental connection between the Golosov localization effect and non-ergodicity in the sense of the disparity between ensemble-averaged MSD and time-averaged MSD.

Statistical physics

Peter Sollich
Section Editor

Statistical physics encompasses the theory of many interacting entities. Originally founded on the description of states of matter comprising atoms and molecules, the theory now extends to the description of active objects and driven systems.

The development of simple mathematical models to elucidate emergent macroscopic behaviour has gone hand in hand with the development of new mathematical techniques for their solution. Building on our understanding of equilibrium states and phase transitions, the grand challenge now lies in the description of nonequilibrium states and myriad associated out-of-equilibrium phenomena.

JPhysA is a leading vehicle for mathematical and theoretical progress in the field.

This selection of articles displays high-quality work in areas of current activity. These include quantum integrable models (M Brockmann, J De Nardis, B Wouters and J-S Caux, 2014 *J. Phys. A: Math. Theor.* **47** 145003), quantum quenches (Leda Bucciantini, Márton Kormos and Pasquale Calabrese, 2014 *J. Phys. A: Math. Theor.* **47** 175002) and neural networks (Adriano Barra, Giuseppe Genovese, Francesco Guerra and Daniele Tantari, 2014 *J. Phys. A: Math. Theor.* **47** 155002).

Stochastic processes are well represented in this collection, in articles including Juraj Szavits-Nossan, Martin R Evans and Satya N Majumdar, 2014 *J. Phys. A: Math. Theor.* **47** 455004; Philipp Thomas, Christian Fleck Ramon Grima and Nikola Popović, 2014 *J. Phys. A: Math. Theor.* **47** 455007 and Roland Bauerschmidt, Wojciech de Roeck and Jürg Fröhlich, 2014 *J. Phys. A: Math. Theor.* **47** 275003.

These and other contributions listed overleaf point to the vitality of the field and the continuing leading role of JPhysA.

STATISTICAL PHYSICS

Adriano Barra

About a solvable mean field model of a Gaussian spin glass

Adriano Barra, Giuseppe Genovese, Francesco Guerra and Daniele Tantari

2014 *J. Phys. A: Math. Theor.* **47** 155002

In a series of papers, we have studied a modified Hopfield model of a neural network, with learned words characterized by a Gaussian distribution. The model can be represented as a bipartite spin glass, with one party described by dichotomic Ising spins, and the other party by continuous spin variables, with an *a priori* Gaussian distribution. By application of standard interpolation methods, we have found it useful to compare the neural network model (bipartite) from one side, with two spin glass models, each monopartite, from the other side. Of these, the first is the usual Sherrington–Kirkpatrick model, the second is a spin glass model, with continuous spins and inbuilt highly nonlinear smooth cut-off interactions. This model is an invaluable laboratory for testing all techniques which have been useful in the study of spin glasses. The purpose of this paper is to give a synthetic description of the most peculiar aspects, by stressing the necessary novelties in the treatment. In particular, it will be shown that the control of the infinite volume limit, according to the well-known Guerra–Toninelli strategy, requires in addition one to consider the involvement of the cut-off interaction in the interpolation procedure. Moreover, the control of the ergodic region, the annealed case, cannot be directly achieved through the standard application of the Borel–Cantelli lemma, but requires previous modification of the interaction. This remark could find useful application in other cases. The replica symmetric expression for the free energy can be easily reached through a suitable version of the doubly stochastic interpolation technique. However, this model shares the unique property that the fully broken replica symmetry ansatz can be explicitly calculated. A very simple sum rule connects the general expression of the fully broken free energy trial function with the replica symmetric one. The definite sign of the error term shows that the replica solution is optimal. Then, we follow a deep strategy developed by Talagrand, to conclude that the replica symmetric ansatz not only gives a bound for the free energy, which cannot be improved by symmetry breaking, but gives also the true value for the free energy. For the sake of completeness, we study also the fluctuations of the (centered and rescaled) overlaps, by following a method similar to that exploited for the Sherrington–Kirkpatrick model, in the replica symmetric region.

STATISTICAL PHYSICS

Winny O'Kelly de Galway

Low temperature behavior of nonequilibrium multilevel systems

Christian Maes, Karel Netočný and Winny O'Kelly de Galway

2014 *J. Phys. A: Math. Theor.* **47** 035002

In this paper, we give a low temperature formula for the stationary occupations in Markovian systems away from detailed balance. Two applications are discussed, one to determine the direction of the ratchet current and one on population inversion. Both of these applications can take advantage of low temperatures to improve the gain and typical nonequilibrium features. Our new formula brings to the foreground the importance of kinetic aspects in terms of reactivities for deciding the levels with highest occupation and, thus, gives a detailed quantitative meaning to Landauer's blowtorch theorem at low temperatures.

Michael Brockmann

A Gaudin-like determinant for overlaps of Néel and XXZ Bethe states

M Brockmann, J De Nardis, B Wouters and J-S Caux

2014 *J. Phys. A: Math. Theor.* **47** 145003

We derive a determinant expression for overlaps of Bethe states of the XXZ spin chain with the Néel state, the ground state of the system in the antiferromagnetic Ising limit. Our formula, of determinant form, is valid for generic system size. Interestingly, it is remarkably similar to the well-known Gaudin formula for the norm of Bethe states, and to another recently-derived overlap formula appearing in the Lieb–Liniger model.

Rodney J Baxter

The τ_2 model and parafermions

R J Baxter

2014 *J. Phys. A: Math. Theor.* **47** 315001

Paul Fendley has recently found a 'parafermionic' way to diagonalize a simple solvable Hamiltonian associated with the chiral Potts model. Here we indicate how this method generalizes to the τ_2 model with open boundaries and make some comments.

STATISTICAL PHYSICS

Persistence of random walk records

E Ben-Naim and P L Krapivsky

2014 *J. Phys. A: Math. Theor.* **47** 255002

We study records generated by Brownian particles in one dimension. Specifically, we investigate an ordinary random walk and define the record as the maximal position of the walk. We compare the record of an individual random walk with the mean record, obtained as an average over infinitely many realizations. We term the walk 'superior' if the record is always above average, and conversely, the walk is said to be 'inferior' if the record is always below average. We find that the fraction of superior walks, S, decays algebraically with time, $S \sim t^{-\beta}$, in the limit $t \to \infty$, and that the persistence exponent is nontrivial, $\beta = 0.382258\ldots$. The fraction of inferior walks, I, also decays as a power law, $I \sim t^{-\alpha}$, but the persistence exponent is smaller, $\alpha = 0.241608\ldots$. Both exponents are roots of transcendental equations involving the parabolic cylinder function. To obtain these theoretical results, we analyze the joint density of superior walks with a given record and position, while for inferior walks it suffices to study the density as a function of position.

Fluctuations in a kinetic transport model for quantum friction

Roland Bauerschmidt, Wojciech de Roeck and Jürg Fröhlich

2014 *J. Phys. A: Math. Theor.* **47** 275003

We study a linear Boltzmann equation describing the effective dynamics of a quantum particle moving through an ideal Bose gas at zero temperature. The particle emits sound waves into the Bose gas, a process causing it to slow down. This mechanism for friction is akin to one exhibited by a charged particle travelling through an optically dense medium at a speed larger than the speed of light and hence emitting Čerenkov radiation. We study the spatial trajectory of the Markov process corresponding to the Boltzmann equation and prove that this process converges to a multiple of Brownian motion after an exponential rescaling of time. We also show that the asymptotic position of the particle remains finite on average, but that its absolute value diverges logarithmically. Our unusual exponential rescaling of time is appropriate for a description of friction at zero temperature and is distinct from diffusive scaling appropriate at positive temperature. It is also shown that if random fluctuations of the particle position were neglected the latter would turn out to diverge.

STATISTICAL PHYSICS

Leda Bucciantini

Quantum quenches from excited states in the Ising chain

Leda Bucciantini, Márton Kormos and Pasquale Calabrese

2014 *J. Phys. A: Math. Theor.* **47** 175002

We consider the non-equilibrium dynamics after a sudden quench of the magnetic field in the transverse field Ising chain starting from excited states of the pre-quench Hamiltonian. We prove that stationary values of local correlation functions can be described by the generalized Gibbs ensemble. Then we study the full time evolution of the transverse magnetization by means of stationary phase methods. The equal-time two-point longitudinal correlation function is analytically derived for a particular class of excited states for quenches within the ferromagnetic phase, and studied numerically in general. The full time dependence of the entanglement entropy of a block of spins is also obtained analytically for the same class of states and for arbitrary quenches.

Fundamentals of higher order stochastic equations

P D Drummond

2014 *J. Phys. A: Math. Theor.* **47** 335001

Stochastic processes that correspond to equations of arbitrary differential order are obtained. We show that stochastic paths in a complex extension of the original phase-space allow an implementation of such higher-order derivative terms. The resulting stochastic process is equivalent to the original partial differential equation in the sense of having equivalent analytic moments. However, the correspondence has unusual properties. Only the analytic moments are convergent, while non-analytic moments such as the complex moduli are non-convergent. These results unify previous approaches that transform higher-derivative equations into probabilistic stochastic equations. Larger ensembles are required as time-steps are reduced, giving these equations unusual convergence properties. This type of process is relevant to the question of how to obtain stochastic quantum simulations using the Wigner representation.

STATISTICAL PHYSICS


Izaak Neri

On the equivalence of Ising models on ‘small-world’ networks and LDPC codes on channels with memory

Izaak Neri and Nikos S Skantzos

2014 *J. Phys. A: Math. Theor.* **47** 385002

We demonstrate the equivalence between thermodynamic observables of Ising spin-glass models on small-world lattices and the decoding properties of error-correcting low-density parity-check codes on channels with memory. In particular, the self-consistent equations for the effective field distributions in the spin-glass model within the replica symmetric ansatz are equivalent to the density evolution equations for Gilbert–Elliott channels. This relationship allows us to present a belief-propagation decoding algorithm for finite-state Markov channels and to compute its performance at infinite block lengths from the density evolution equations. We show that loss of reliable communication corresponds to a first order phase transition from a ferromagnetic phase to a paramagnetic phase in the spin glass model. The critical noise levels derived for Gilbert–Elliott channels are in very good agreement with existing results in coding theory. Furthermore, we use our analysis to derive critical noise levels for channels with both memory and asymmetry in the noise. The resulting phase diagram shows that the combination of asymmetry and memory in the channel allows for high critical noise levels: in particular, we show that successful decoding is possible at any noise level of the bad channel when the good channel is good enough. Theoretical results at infinite block lengths using density evolution equations aree compared with average error probabilities calculated from a practical implementation of the corresponding decoding algorithms at finite block lengths.

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STATISTICAL PHYSICS

Juraj Szavits-Nossan

Condensation transition in joint large deviations of linear statistics

Juraj Szavits-Nossan, Martin R Evans and Satya N Majumdar

2014 *J. Phys. A: Math. Theor.* **47** 455004

Real space condensation is known to occur in stochastic models of mass transport in the regime in which the globally conserved mass density is greater than a critical value. It has been shown within models with factorized stationary states that the condensation can be understood in terms of sums of independent and identically distributed random variables: these exhibit condensation when they are conditioned to a large deviation of their sum. It is well understood that the condensation, whereby one of the random variables contributes a finite fraction to the sum, occurs only if the underlying probability distribution (modulo exponential) is heavy-tailed, i.e. decaying slower than exponential. Here we study a similar phenomenon in which condensation is exhibited for non-heavy-tailed distributions, provided random variables are additionally conditioned on a large deviation of certain linear statistics. We provide a detailed theoretical analysis explaining the phenomenon, which is supported by Monte Carlo simulations (for the case where the additional constraint is the sample variance) and demonstrated in several physical systems. Our results suggest that the condensation is a generic phenomenon that pertains to both typical and rare events.

Philipp Thomas

System size expansion using Feynman rules and diagrams

Philipp Thomas, Christian Fleck, Ramon Grima and Nikola Popović

2014 *J. Phys. A: Math. Theor.* **47** 455007

Few analytical methods exist for quantitative studies of large fluctuations in stochastic systems. In this article, we develop a simple diagrammatic approach to the chemical master equation that allows us to calculate multi-time correlation functions which are accurate to any desired order in van Kampen's system size expansion. Specifically, we present a set of *Feynman rules* from which this diagrammatic perturbation expansion can be constructed algorithmically. We then apply the methodology to derive in closed form the leading order corrections to the linear noise approximation of the intrinsic noise power spectrum for general biochemical reaction networks. Finally, we illustrate our results by describing noise-induced oscillations in the Brusselator reaction scheme which are not captured by the common linear noise approximation.

Chaotic and complex systems

Arkady Pikovsky
Section Editor

Furious development of nonlinear physics in the last decades of the 20th century, due to progress in such fields as chaos, nonlinear pattern formation and turbulence along with the non-trivial role of noise in nonlinear systems, resulted in establishing it, together with statistical physics, as a major methodological interdisciplinary approach to the study of classical dynamical phenomena.

This success story continues in the field of complexity science, fulfilling the expectation of Stephen Hawking, who in 2000 said: “I think the next century will be the century of complexity.”

JPhysA offers an optimal platform both for presenting theoretically and mathematically oriented contributions to the basics of nonlinear and complex systems and for publishing diverse applied studies in this truly interdisciplinary field.

Excellent examples are the recent review articles “Revealing networks from dynamics: an introduction” by Timme and Casadiego (2014 *J. Phys. A: Math. Theor.* **47** 343001), “Time scale of diffusion in molecular and cellular biology” by Holcman and Schuss (2014 *J. Phys. A: Math. Theor.* **47** 173001) and “Nonlinear lattice waves in heterogeneous media” Laptyeva, Ivanchenko and Flach (2014 *J. Phys. A: Math. Theor.* **47** 493001) along with the original research presented overleaf.

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CHAOTIC AND COMPLEX SYSTEMS

Moo Young Choi

Discriminating between Weibull distributions and log-normal distributions emerging in branching processes

Segun Goh, H W Kwon and M Y Choi

2014 *J. Phys. A: Math. Theor.* **47** 225101

We consider the Yule-type multiplicative growth and division process, and describe the ubiquitous emergence of Weibull and log-normal distributions in a single framework. With the help of the integral transform and series expansion, we show that both distributions serve as asymptotic solutions of the time evolution equation for the branching process. In particular, the maximum likelihood method is employed to discriminate between the emergence of the Weibull distribution and that of the log-normal distribution. Further, the detailed conditions for the distinguished emergence of the Weibull distribution are probed. It is observed that the emergence depends on the manner of the division process for the two different types of distribution. Numerical simulations are also carried out, confirming the results obtained analytically.

Daniel Waltner

Transmission through a noisy network

Daniel Waltner and Uzy Smilansky

2014 *J. Phys. A: Math. Theor.* **47** 355101

Quantum graphs with leads to infinity serve as convenient models for studying various aspects of systems which are usually attributed to chaotic scattering. They are also studied in several experimental systems and practical applications. In the present manuscript we investigate the effect of a time dependent random noise on the transmission of such graphs, and in particular on the resonances which dominate the scattering observable such as the transmission and reflection intensities. We model the noise by a potential $\alpha\delta(x-(x_0+\gamma(t)))$ localized at an arbitrary point x_0 on any of the graph bonds, that fluctuates in time as a Brownian particle bounded in a harmonic potential described by the Ornstein–Uhlenbeck statistics. These statistics, which bind the Brownian motion within a finite interval, enable the use of a second order time-dependent perturbation theory, which can be applied whenever the strength parameter α is sufficiently small. The theoretical framework will be explained in full generality, and will be explicitly solved for a simple, yet nontrivial example.

CHAOTIC AND COMPLEX SYSTEMS

Vassili Gelfreich

Oscillating mushrooms: adiabatic theory for a non-ergodic system

V Gelfreich, V Rom-Kedar and D Turaev

2014 *J. Phys. A: Math. Theor.* **47** 395101

Can elliptic islands contribute to sustained energy growth as parameters of a Hamiltonian system slowly vary with time? In this paper we show that a mushroom billiard with a periodically oscillating boundary accelerates the particle inside it exponentially fast. We provide an estimate for the rate of acceleration. Our numerical experiments corroborate the theory. We suggest that a similar mechanism applies to general systems with mixed phase space.

Mark Dennis

Geometry and scaling of tangled vortex lines in three-dimensional random wave fields

A J Taylor and M R Dennis

2014 *J. Phys. A: Math. Theor.* **47** 465101

The short- and long-scale behaviour of tangled wave vortices (nodal lines) in random three-dimensional (3D) wave fields is studied via computer experiment. The zero lines are tracked in numerical simulations of periodic superpositions of 3D complex plane waves. The probability distribution of local geometric quantities such as curvature and torsion are compared to previous analytical and new Monte Carlo results from the isotropic Gaussian random wave model. We further examine the scaling and self-similarity of tangled wave vortex lines individually and in the bulk, drawing comparisons with other physical systems of tangled filaments.

Mathematical physics

Atsuo Kuniba
Section Editor

In this section, we welcome papers in which the ideas, techniques and applications flow between theoretical physics and mathematics in either direction. We had the pleasure of publishing excellent papers in 2014 with some of the most interesting contributions listed in this Highlights collection. Here is a brief summary from them.

The octahedron recurrence relation is a bilinear difference equation for a scalar field living on the cubic lattice. It is an infinite rank case of the type A T-system and plays a central role in a number of combinatorial problems including alternating sign matrices, Littlewood-Richardson coefficients, cluster algebras, etc. Di Francesco and Soto-Garrido (*J. Phys. A: Math. Theor.* **47** 285204) utilized its connection to the dimer covering of the Aztec diamond and constructed the solution for the m-toroidal boundary condition.

The thermodynamic limit of the solution yields the exact locus of the frozen, temperate and facet domains distinguished by the local entropy of the dimer configurations.

Rogue waves are outstanding nonlinear waves which ‘come from nowhere and disappear with no trace’. They have been studied with increasing interest due to possible links to freak waves in the ocean, high-intensity waves in optical fibres and so on. Most analytic results on rogue waves have been concerned with continuous wave equations.

Ohta and Yang (*J. Phys. A: Math. Theor.* **47** 255201) studied the nonlinear lattice known as the Ablowitz-Ladik equation in either focusing and defocusing regime and constructed rogue wave solutions of arbitrary order explicitly. It is shown that rogue waves exist even in the defocusing case and that the peaks in the higher order solutions exhibit rich patterns with individual amplitudes.

Exceptional orthogonal polynomials are families of orthogonal polynomials with a finite number of gaps in their degree sequence that nevertheless span a complete basis of the corresponding Hilbert spaces. They have found a variety of applications in superintegrable systems, Dirac operators coupled to external fields, entropy measures in quantum information theory and so on. Gómez-Ullate *et al.* (*J. Phys. A: Math. Theor.* **47** 015203) established several fundamental properties of exceptional Hermite polynomials in relation to the classification of the rational extension of the quantum harmonic oscillator.

MATHEMATICAL PHYSICS

David Gómez-Ullate

Rational extensions of the quantum harmonic oscillator and exceptional Hermite polynomials

David Gómez-Ullate, Yves Grandati and Robert Milson

2014 *J. Phys. A: Math. Theor.* **47** 015203

We prove that every rational extension of the quantum harmonic oscillator that is exactly solvable by polynomials is monodromy free, and therefore can be obtained by applying a finite number of state-deleting Darboux transformations on the harmonic oscillator. Equivalently, every exceptional orthogonal polynomial system of Hermite type can be obtained by applying a Darboux–Crum transformation to the classical Hermite polynomials. Exceptional Hermite polynomial systems only exist for even codimension $2m$, and they are indexed by the partitions λ of m. We provide explicit expressions for their corresponding orthogonality weights and differential operators and a separate proof of their completeness. Exceptional Hermite polynomials satisfy a $2l + 3$ recurrence relation where l is the length of the partition λ. Explicit expressions for such recurrence relations are given.

Peter J Forrester

Probability of all eigenvalues real for products of standard Gaussian matrices

Peter J Forrester

2014 *J. Phys. A: Math. Theor.* **47** 065202

With $\{X_i\}$ independent $N \times N$ standard Gaussian random matrices, the probability $P^{Pm}_{N,N}$ that all eigenvalues are real for the matrix product $P_m = X_m X_{m-1} \cdots X_1$ is expressed in terms of an $N/2 \times N/2$ (N even) and $(N + 1)/2 \times (N + 1)/2$ (N odd) determinant. The entries of the determinant are certain Meijer G-functions. In the case $m = 2$ high precision computation indicates that the entries are rational multiples of π^2, with the denominator a power of 2, and that to leading order in N $P^{Pm}_{N,N}$ decays as $(\pi/4)N^2/2$. We are able to show that for general m and large N, $P^{Pm}_{N,N} \sim b_m^{N^2}$ with an explicit b_m. An analytic demonstration that $P^{Pm}_{N,N} \to 1$ as m $\to \infty$ is given.

MATHEMATICAL PHYSICS

General rogue waves in the focusing and defocusing Ablowitz–Ladik equations

Yasuhiro Ohta and Jianke Yang

2014 *J. Phys. A: Math. Theor.* **47** 255201

General rogue waves in the focusing and defocusing Ablowitz–Ladik equations are derived by the bilinear method. In the focusing case, it is shown that rogue waves are always bounded. In addition, fundamental rogue waves reach peak amplitudes which are at least three times that of the constant background, and higher-order rogue waves can exhibit patterns such as triads and circular arrays with different individual peaks. In the defocusing case, it is shown that rogue waves also exist. In addition, these waves can blow up to infinity in finite time.

François Gay-Balmaz

Darryl D Holm

Tudor S Ratiu

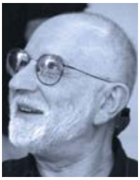
Integrable G-strands on semisimple Lie groups

François Gay-Balmaz, Darryl D Holm and Tudor S Ratiu

2014 *J. Phys. A: Math. Theor.* **47** 075201

The present paper derives systems of partial differential equations that admit a quadratic zero curvature representation for an arbitrary real semisimple Lie algebra. It also determines the general form of Hamilton's principles and Hamiltonians for these systems, and analyzes the linear stability of their equilibrium solutions in the examples of $\mathfrak{so}(3)$and $\mathfrak{sl}(2, \mathbb{R})$.

MATHEMATICAL PHYSICS



Michael V Berry

Pragya Shukla

Superadiabatic optical forces on a dipole: exactly solvable model for a vortex field

M V Berry and Pragya Shukla

2014 *J. Phys. A: Math. Theor.* **47** 125201

The forces exerted by light on a small particle are modified by the particle's motion, giving a series of superadiabatic corrections to the lowest-order approximation in which the motion is neglected. The correction forces can be calculated recursively for an electric dipole modelled as a damped oscillator. In lowest order, there is, as is known, a non-potential though non-dissipative 'curl force', in addition to the familiar gradient force. In the next order, there are forces of geometric magnetism and friction, related to the geometric phase 2-form and the metric of the driving field. For the paraxial field of an optical vortex, the hierarchy of superadiabatic forces can be calculated explicitly, revealing a four-sheeted Riemann surface on which fast and slow dynamics are connected. This leads to an exact 'slow manifold', on which the dipole is driven without oscillations by the same forces as in the first two adiabatic orders, but with frequency-renormalized strengths.

Thomas L Curtright

Cosmas K Zachos

Branched Hamiltonians and supersymmetry

T L Curtright and C K Zachos

2014 *J. Phys. A: Math. Theor.* **47** 145201

Some examples of branched Hamiltonians are explored both classically and in the context of quantum mechanics, as recently advocated by Shapere and Wilczek. These are in fact cases of switchback potentials, albeit in momentum space, as previously analyzed for quasi-Hamiltonian chaotic dynamical systems in a classical setting, and as encountered in analogous renormalization group flows for quantum theories which exhibit RG cycles. A basic two-worlds model, with a pair of Hamiltonian branches related by supersymmetry, is considered in detail.

MATHEMATICAL PHYSICS

Philippe Di Francesco

Arctic curves of the octahedron equation

Philippe Di Francesco and Rodrigo Soto-Garrido

2014 *J. Phys. A: Math. Theor.* **47** 285204

We study the octahedron relation (also known as the A_∞ T-system), obeyed in particular by the partition function for dimer coverings of the Aztec Diamond graph. For a suitable class of doubly periodic initial conditions, we find exact solutions with a particularly simple factorized form. For these, we show that the density function that measures the average dimer occupation of a face of the Aztec graph, obeys a system of linear recursion relations with periodic coefficients. This allows us to explore the thermodynamic limit of the corresponding dimer models and to derive exact 'arctic' curves separating the various phases of the system.

Alexandre Lazarescu

Vincent Pasquier

Bethe Ansatz and Q-operator for the open ASEP

Alexandre Lazarescu and Vincent Pasquier

2014 *J. Phys. A: Math. Theor.* **47** 295202

In this paper, we look at the asymmetric simple exclusion process with open boundaries with a current-counting deformation. We construct a two-parameter family of transfer matrices which commute with the deformed Markov matrix of the system. We show that these transfer matrices can be factorized into two commuting matrices with one parameter each, which can be identified with Baxter's Q-operator, and that for certain values of the product of those parameters, they decompose into a sum of two commuting matrices, one of which is the usual one-parameter transfer matrix for a given dimension of the auxiliary space. Using this, we find the T–Q equation for the open ASEP, and, through functional Bethe Ansatz techniques, we obtain an exact expression for the dominant eigenvalue of the deformed Markov matrix.

MATHEMATICAL PHYSICS

Germán Sierra

The Riemann zeros as energy levels of a Dirac fermion in a potential built from the prime numbers in Rindler spacetime

Germán Sierra

2014 *J. Phys. A: Math. Theor.* **47** 325204

We construct a Hamiltonian H_R whose discrete spectrum contains, in a certain limit, the Riemann zeros. H_R is derived from the action of a massless Dirac fermion living in a domain of Rindler spacetime, in 1 + 1 dimensions, which has a boundary given by the world line of a uniformly accelerated observer. The action contains a sum of delta function potentials that can be viewed as partially reflecting moving mirrors. An appropriate choice of the accelerations of the mirrors, provide *primitive* periodic orbits that are associated with the prime numbers *p*, whose periods, as measured by the observer's clock, are $\log p$. Acting on the chiral components of the fermion $\chi_{\mp}$, H_R becomes the Berry–Keating Hamiltonian $\pm(x\hat{p}+\hat{p}x)/2$, where x is identified with the Rindler spatial coordinate and $\hat{p}$ with the conjugate momentum. The delta function potentials give the matching conditions of the fermion wave functions on both sides of the mirrors. There is also a phase shift $e^{i\Theta}$ for the reflection of the fermions at the boundary where the observer sits. The eigenvalue problem is solved by transfer matrix methods in the limit where the reflection amplitudes become infinitesimally small. We find that, for generic values of ϑ, the spectrum is a continuum where the Riemann zeros are missing, as in the adelic Connes model. However, for some values of ϑ, related to the phase of the zeta function, the Riemann zeros appear as discrete eigenvalues that are immersed in the continuum. We generalize this result to the zeros of Dirichlet *L*-functions, which are associated to primitive characters, that are encoded in the reflection coefficients of the mirrors. Finally, we show that the Hamiltonian associated to the Riemann zeros belongs to class AIII, or chiral GUE, of the Random Matrix Theory.

On the *S*-matrix of Schrödinger operators with non-symmetric zero-range potentials

P A Cojuhari, A Grod and S Kuzhel

2014 *J. Phys. A: Math. Theor.* **47** 315201

Non-self-adjoint Schrödinger operators $A_{\mathfrak{T}}$ which correspond to non-symmetric zero-range potentials are investigated. We show that various properties of $A_{\mathfrak{T}}$ (eigenvalues, exceptional points, spectral singularities and the property of similarity to a self-adjoint operator) are completely determined by poles of the corresponding *S*-matrix.

MATHEMATICAL PHYSICS

Mario Kieburg

Universal distribution of Lyapunov exponents for products of Ginibre matrices

Gernot Akemann, Zdzislaw Burda and Mario Kieburg

2014 *J. Phys. A: Math. Theor.* **47** 395202

Starting from exact analytical results on singular values and complex eigenvalues of products of independent Gaussian complex random $N \times N$ matrices, also called the Ginibre ensemble, we rederive the Lyapunov exponents for an infinite product. We show that for a large number t of product matrices, the distribution of each Lyapunov exponent is normal, and we compute its t-dependent variance as well as corrections in a large-t expansion. Originally Lyapunov exponents are defined for the singular values of the product matrix that represents a linear time evolution. Surprisingly a similar construction for the moduli of the complex eigenvalues yields the very same exponents and normal distributions to leading order. We discuss a general mechanism for 2×2 matrices showing why the singular values and the radii of complex eigenvalues collapse onto the same value in the large-t limit. We thereby rederive Newman's triangular law which has a simple interpretation as the radial density of complex eigenvalues in the circular law, and study the commutativity of the two limits $t \to \infty$ and $N \to \infty$ on the global scale and on the local scale. We show as a mathematical byproduct that a particular asymptotic expansion of a Meijer G-function with large index leads to a Gaussian.

Fumitaka Yura

Solitons with a nested structure over finite fields

Fumitaka Yura

2014 *J. Phys. A: Math. Theor.* **47** 325201

We propose a solitonic dynamical system over finite fields that may be regarded as an analogue of the box–ball systems. The one-soliton solutions of the system, which have nested structures similar to fractals, are also proved. The solitonic system in this paper is described by polynomials, which seems to be novel. Furthermore, in spite of such complex internal structures, numerical simulations exhibit stable propagations before and after collisions among multiple solitons, preserving their patterns.

Quantum mechanics and quantum information theory

Otfried Gühne
Section Editor

The quantum mechanics and quantum information theory section of the journal publishes a broad spectrum of papers, from research that considers the foundational and conceptual ideas underlying quantum mechanics to articles that discuss aspects of quantum information as diverse as entanglement measures, quantum communication and cryptography.

This collection features papers from across this spectrum: these include Datta and Leditzky (2014 *J. Phys. A: Math. Theor.* **47** 045304) who work on Rényi entropy, which has diverse applications across quantum information theory and Augusiak, Bae, Tura and Lewenstein (2014 *J. Phys. A: Math. Theor.* **47** 065301) who discuss optimality of entanglement witnesses. Dall'ArnoBuscemi and Ozawa (2014 *J. Phys. A: Math. Theor.* **47** 235302) find tight upper and lower bounds on the accessible information of quantum states and Girard, Gour and Friedland (2014 *J. Phys. A: Math. Theor.* **47** 505302) seek solutions to convex optimisation problems, which has implications throughout quantum information theory.

We were very pleased to publish a special issue to mark 50 years of Bell's theorem. The contributions in this issue range from foundational issues on Bell's theorem to applications of entanglement in quantum metrology. The papers by Maudlin (*J. Phys. A: Math. Theor.* **47** 424010) and Werner (*J. Phys. A: Math. Theor.* **47** 424011) may serve as representatives for a foundational debate about what it was that Bell's theorem proved.

Teiko Heinosaari

Jukka Kiukas

Coexistence of effects from an algebra of two projections

Teiko Heinosaari, Jukka Kiukas and Daniel Reitzner

2014 *J. Phys. A: Math. Theor.* **47** 225301

The coexistence relation of quantum effects is a fundamental structure, describing those pairs of experimental events that can be implemented in a single setup. Only in the simplest case of qubit effects is an analytic characterization of coexistent pairs known. We generalize the qubit coexistence characterization to all pairs of effects in arbitrary dimensions that belong to the von Neumann algebra generated by two projections. We demonstrate the presented mathematical machinery by several examples, and show that it covers physically relevant classes of effect pairs.

QUANTUM MECHANICS AND QUANTUM INFORMATION THEORY

Remigiusz Augusiak

Checking the optimality of entanglement witnesses: an application to structural physical approximations

R Augusiak, J Bae, J Tura and M Lewenstein

2014 *J. Phys. A: Math. Theor.* **47** 065301

In 2008, the conjecture that structural physical approximations (SPAs) to optimal entanglement witnesses are separable states (in general unnormalized) was posed. In an attempt to disprove it, Ha and Kye (2012 arXiv:1210.1088v3) proposed a decomposable entanglement witness, whose SPA is entangled, and argued that it is optimal. In this paper, which is based on a comment to the latter work (Augusiak *et al* 2013 arXiv:1304.2040v1), we show, both analytically and numerically, that this entanglement witness is not optimal, and as such it is not a counterexample to the conjecture. To this end, we make use of a method for checking the optimality of entanglement witnesses developed already in Lewenstein *et al* (2000 *Phys. Rev. A* **62** 052310), however, hardly exploited so far in the literature.

A limit of the quantum Rényi divergence

Nilanjana Datta and Felix Leditzky

2014 *J. Phys. A: Math. Theor.* **47** 045304

Recently, an interesting quantity called the quantum Rényi divergence (or 'sandwiched' Rényi relative entropy) was defined for pairs of positive semi-definite operators ρ and σ. It depends on a parameter α and acts as a parent quantity for other relative entropies which have important operational significance in quantum information theory: the quantum relative entropy and the min- and max-relative entropies. There is, however, another relative entropy, called the 0-relative Rényi entropy, which plays a key role in the analysis of various quantum information-processing tasks in the one-shot setting. We prove that the 0-relative Rényi entropy is obtainable from the quantum Rényi divergence only if ρ and σ have equal supports. This, along with existing results in the literature, suggests that it suffices to consider two essential parent quantities from which operationally relevant entropic quantities can be derived—the quantum Rényi divergence with parameter $\alpha \geq 1/2$, and the α-relative Rényi entropy with $\alpha \in [0, 1)$.

QUANTUM MECHANICS AND QUANTUM INFORMATION THEORY

Nikolas P Breuckmann

Space-time circuit-to-Hamiltonian construction and its applications

Nikolas P Breuckmann and Barbara M Terhal

2014 *J. Phys. A: Math. Theor.* **47** 195304

The circuit-to-Hamiltonian construction translates dynamics (a quantum circuit and its output) into statics (the groundstate of a circuit Hamiltonian) by explicitly defining a quantum register for a clock. The standard Feynman–Kitaev construction uses one global clock for all qubits while we consider a different construction in which a clock is assigned to each interacting qubit. This makes it possible to capture the spatio-temporal structure of the original quantum circuit into features of the circuit Hamiltonian. The construction is inspired by the original two-dimensional interacting fermion model in Mizel *et al* (2001 *Phys. Rev. A* **63** 040302). We prove that for one-dimensional quantum circuits the gap of the circuit Hamiltonian is appropriately lowerbounded so that the applications of this construction for quantum Merlin–Arthur (and partially for quantum adiabatic computation) go through. For one-dimensional quantum circuits, the dynamics generated by the circuit Hamiltonian corresponds to the diffusion of a string around the torus.

Mark W Girard

On convex optimization problems in quantum information theory

Mark W Girard, Gilad Gour and Shmuel Friedland

2014 *J. Phys. A: Math. Theor.* **47** 505302

Convex optimization problems arise naturally in quantum information theory, often in terms of minimizing a convex function over a convex subset of the space of hermitian matrices. In most cases, finding exact solutions to these problems is usually impossible. As inspired by earlier investigations into the relative entropy of entanglement (REE) (Miranowicz and Ishizaka 2008 *Phys. Rev. A* **78** 032310), we introduce a general method to solve the converse problem rather than find explicit solutions. That is, given a matrix in a convex set, we determine a family of convex functions that are minimized at this point. This method allows us find explicit formulae for the REE and the Rains bound, two well-known upper bounds on the distillable entanglement, and yields interesting information about these quantities, such as the fact that they coincide in the case where at least one subsystem of a multipartite state is a qubit.

QUANTUM MECHANICS AND QUANTUM INFORMATION THEORY

Michele Dall'Arno

Tight bounds on accessible information and informational power

Michele Dall'Arno, Francesco Buscemi and Masanao Ozawa

2014 *J. Phys. A: Math. Theor.* **47** 235302

The accessible information quantifies the amount of classical information that can be extracted from an ensemble of quantum states. Analogously, the informational power quantifies the amount of classical information that can be extracted by a quantum measurement. For both quantities, we provide upper and lower bounds that depend only on the dimension of the system, and we prove their tightness. In the case of symmetric informationally complete (SIC) ensembles and measurements, stronger bounds are provided and their tightness proved for qubits and qutrits. From our upper bounds, we notice, perhaps surprisingly, that the statistics generated by SIC ensembles or measurements in arbitrary dimension, though optimal for tomographic purposes, in fact never contain more than just one bit of information, the rest being constituted by completely random bits. On the other hand, from our lower bounds, we obtain an explicit strategy beating the so-called pretty-good one for the extraction of mutual information in the case of SIC ensembles and measurements.

Hypercontractivity of quasi-free quantum semigroups

Kristan Temme, Fernando Pastawski and Michael J Kastoryano

2014 *J. Phys. A: Math. Theor.* **47** 405303

Hypercontractivity of a quantum dynamical semigroup has strong implications for its convergence behavior and entropy decay rate. A logarithmic Sobolev inequality and the corresponding logarithmic Sobolev constant can be inferred from the semigroup's hypercontractive norm bound. We consider completely-positive quantum mechanical semigroups described by a Lindblad master equation. To prove the norm bound, we follow an approach which has its roots in the study of classical rate equations. We use interpolation theorems for non-commutative $\mathbb{L}_p$ spaces to obtain a general hypercontractive inequality from a particular $p \to q$-norm bound. Then, we derive a bound on the $2 \to 4$-norm from an analysis of the block diagonal structure of the semigroup's spectrum. We show that the dynamics of an N-qubit graph state Hamiltonian weakly coupled to a thermal environment is hypercontractive. As a consequence this allows for the efficient preparation of graph states in time $\mathrm{poly}(\log(N))$ by coupling at sufficiently low temperature. Furthermore, we extend our results to gapped Liouvillians arising from a weak linear coupling of a free-fermion systems.

QUANTUM MECHANICS AND QUANTUM INFORMATION THEORY


Matthew S Leifer


Robert W Spekkens

A Bayesian approach to compatibility, improvement, and pooling of quantum states

M S Leifer and Robert W Spekkens

2014 *J. Phys. A: Math. Theor.* **47** 275301

In approaches to quantum theory in which the quantum state is regarded as a representation of knowledge, information, or belief, two agents can assign different states to the same quantum system. This raises two questions: when are such state assignments compatible? And how should the state assignments of different agents be reconciled? In this paper, we address these questions from the perspective of the recently developed conditional states formalism for quantum theory (Leifer M S and Spekkens R W 2013 *Phys. Rev. A* **88** 052310). Specifically, we derive a compatibility criterion proposed by Brun, Finkelstein and Mermin from the requirement that, upon acquiring data, agents should update their states using a quantum generalization of Bayesian conditioning. We provide two alternative arguments for this criterion, based on the objective and subjective Bayesian interpretations of probability theory. We then apply the same methodology to the problem of quantum state improvement, i.e. how to update your state when you learn someone else's state assignment, and to quantum state pooling, i.e. how to combine the state assignments of several agents into a single assignment that accurately represents the views of the group. In particular, we derive a pooling rule previously proposed by Spekkens and Wiseman under much weaker assumptions than those made in the original derivation. All of our results apply to a much broader class of experimental scenarios than have been considered previously in this context.


Kate Blanchfield

Order 3 symmetry in the Clifford hierarchy

Ingemar Bengtsson, Kate Blanchfield, Earl Campbell and Mark Howard

2014 *J. Phys. A: Math. Theor.* **47** 455302

We investigate the action of the first three levels of the Clifford hierarchy on sets of mutually unbiased bases comprising the Ivanovic mutually unbiased base (MUB) and the Alltop MUBs. Vectors in the Alltop MUBs exhibit additional symmetries when the dimension is a prime number equal to 1 modulo 3 and thus the set of all Alltop vectors splits into three Clifford orbits. These vectors form configurations with so-called Zauner subspaces, eigenspaces of order 3 elements of the Clifford group highly relevant to the SIC problem. We identify Alltop vectors as the magic states that appear in the context of fault-tolerant universal quantum computing, wherein the appearance of distinct Clifford orbits implies a surprising inequivalence between some magic states.

QUANTUM MECHANICS AND QUANTUM INFORMATION THEORY


Tim Maudlin

What Bell did

Tim Maudlin

2014 *J. Phys. A: Math. Theor.* **47** 424010

On the 50th anniversary of Bell's monumental 1964 paper, there is still widespread misunderstanding about exactly what Bell proved. This misunderstanding derives in turn from a failure to appreciate the earlier argument of Einstein, Podolsky and Rosen. I retrace the history and logical structure of these arguments in order to clarify the proper conclusion, namely that any world that displays violations of Bell's inequality for experiments done far from one another must be non-local. Since the world we happen to live in displays such violations, actual physics is non-local.

This article is part of a special issue of *Journal of Physics A: Mathematical and Theoretical* devoted to '50 years of Bell's theorem'.

Comment on 'What Bell did'

Reinhard F Werner

2014 *J. Phys. A: Math. Theor.* **47** 424011

According to Maudlin, Bell showed that it is the World which is non-local, and not just some particular theories of it. I argue that this conclusion is arrived at by taking for granted all assumptions of realism or 'classicality'. If these are taken into account the resulting conclusion that 'a classical world which allows for Bell inequality violations must be non-local' is in good agreement with the mainstream perception of Bell's theorem(s).

This article is part of a special issue of *Journal of Physics A: Mathematical and Theoretical* devoted to '50 years of Bell's theorem'.

Field theory and string theory

Arkady Tseytlin
Section Editor

The field theory and string theory section publishes high-quality and significant new results in areas including general methods of quantum field theory, supersymmetric field theories in various dimensions, conformal field theories, integrable theories, string theory and its applications.

It has been another good year for the section with the highlighted papers reporting research from across the section. These include papers on instantons, scattering amplitudes, supersymmetry, logarithmic conformal field theories – an area featured in a recent special issue of the journal – as well as the traditionally strong area of gauge-string duality and integrability, which also featured in a topical review 'Integrability of the AdS_5 x S^5 superstring and its deformations' from Stijn J van Tongeren. We have also seen a number of papers published in the currently active subject of higher spin theories and look forward to an upcoming special issue on localisation techniques in quantum field theories.

I am pleased to announce the new editor for this section will be Joseph Minahan of Uppsala University, Sweden, and his appointment will further strengthen the section as a home for important and high-quality work.

We hope you enjoy these articles and look forward to publishing more high-quality work across the section in 2015.

Thomas Klose

Comments on world-sheet form factors in AdS/CFT

Thomas Klose and Tristan McLoughlin

2014 *J. Phys. A: Math. Theor.* **47** 055401

We study form factors in the light-cone gauge world-sheet theory for strings in $AdS_5 \times S^5$. We perturbatively calculate the two-particle form factor in a closed $\mathfrak{su}(2)$ sector to one-loop in the near-plane-wave limit and to two-loops in the Maldacena–Swanson limit. We also perturbatively solve the functional equation which follows from the form factor axioms for the world-sheet theory and show that the 'minimal' solution correctly reproduces the discontinuities of the perturbative calculations. Finally we propose a prescription, valid for polynomial orders of the inverse world-sheet length, for extracting the finite-volume world-sheet matrix element from the form factors and show that the two-excitation matrix element matches with the thermodynamic limit of the spin–chain description of certain tree-level N=4^SYM^ structure constants.

FIELD THEORY AND STRING THEORY

Martin Heinze
(copyright: Marta Mayer DESY)

Static gauge and energy spectrum of single-mode strings in $AdS_5 \times S^5$

Sergey Frolov, Martin Heinze, George Jorjadze and Jan Plefka

2014 *J. Phys. A: Math. Theor.* **47** 085401

Motivated by the notorious difficulties in determining the first quantum corrections to the spectrum of short strings in $AdS_5 \times S^5$ from first principles, we study closed bosonic strings in this background employing a static gauge. In this gauge the world-sheet Hamiltonian density is constant along the extension of the string and directly proportional to the square of the spacetime energy. We quantize this system in a minisuperspace approach, in which we consider only a single AdS_5 string-mode excitation next to an arbitrary particle like zero-mode contribution in the full $AdS_5 \times S^5$ background. We determine the quantum spectrum using this method to the next-to-next-to-leading order in the large 't Hooft coupling expansion. We argue for an ordering prescription which should arise from supersymmetrization and indeed recover the integrability based predictions for the spectrum of the lightest excitation, dual to the Konishi field scaling dimensions. The higher excitations fail to agree, but this is shown to be a consequence of the string-mode truncation employed. Despite this simple setup, our system reveals intriguing features, such as a close connection to particles in AdS_6, classical integrability and preservation of the isometries of $AdS_5 \times S^5$ at the quantum level.

Alasdair J Routh

All superparticles are BPS

Luca Mezincescu, Alasdair J Routh and Paul K Townsend

2014 *J. Phys. A: Math. Theor.* **47** 175401

The generic action for an N-extended superparticle in D-dimensional Minkowski spacetime is shown to have 'hidden' supersymmetries (related by 'dualities' to the manifest supersymmetries) such that the full supersymmetry algebra is BPS-saturated; the exceptions (which include, trivially, the massless case) are those for which the manifest supersymmetry algebra is already BPS-saturated. Moreover, it is shown that any 'non-BPS' superparticle action is a gauge-fixed version of the 'BPS' superparticle action for which all supersymmetries are manifest. An example is the $N = 1$ massive $D = 10$ superparticle, which actually has $N = 2$ supersymmetry and is equivalent to the action for a D0-brane of IIA superstring theory.

FIELD THEORY AND STRING THEORY

Paul Sutcliffe

A low-dimensional analogue of holographic baryons

Stefano Bolognesi and Paul Sutcliffe

2014 *J. Phys. A: Math. Theor.* **47** 135401

Baryons in holographic QCD correspond to topological solitons in the bulk. The most prominent example is the Sakai–Sugimoto model, where the bulk soliton in the five-dimensional spacetime of AdS-type can be approximated by the flat space self-dual Yang–Mills instanton with a small size. Recently, the validity of this approximation has been verified by comparison with the numerical field theory solution. However, multi-solitons and solitons with finite density are currently beyond numerical field theory computations. Various approximations have been applied to investigate these important issues and have led to proposals for finite density configurations that include dyonic salt and baryonic popcorn. Here we introduce and investigate a low-dimensional analogue of the Sakai–Sugimoto model, in which the bulk soliton can be approximated by a flat space sigma model instanton. The bulk theory is a baby Skyrme model in a three-dimensional spacetime with negative curvature. The advantage of the lower-dimensional theory is that numerical simulations of multi-solitons and finite density solutions can be performed and compared with flat space instanton approximations. In particular, analogues of dyonic salt and baryonic popcorn configurations are found and analysed.

Johannes Broedel
(copyright: Heidi Hostettler)

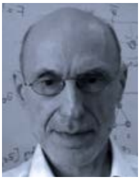
On Yangian-invariant regularization of deformed on-shell diagrams in $\mathcal{N} = 4$ super-Yang–Mills theory

Niklas Beisert, Johannes Broedel and Matteo Rosso

2014 *J. Phys. A: Math. Theor.* **47** 365402

We investigate Yangian invariance of deformed on-shell diagrams with $D = 4$, $\mathcal{N} = 4$ superconformal symmetry. We find that invariance implies a direct relationship between the deformation parameters and the permutation associated with the on-shell graph. We analyse the connection with deformations of scattering amplitudes in $\mathcal{N} = 4$ super-Yang–Mills theory and the possibility of using the deformation parameters as a regulator preserving Yangian invariance. A study of higher-point tree and loop graphs suggests that manifest Yangian invariance of the amplitude requires trivial deformation parameters.

FIELD THEORY AND STRING THEORY



Ivan Kostov

Semi-classical analysis of the inner product of Bethe states

Eldad Bettelheim and Ivan Kostov

2014 *J. Phys. A: Math. Theor.* **47** 245401

We study the inner product of two Bethe states, one of which is taken on-shell, in an inhomogeneous XXX chain in the Sutherland limit, where the number of magnons is comparable with the length L of the chain and the magnon rapidities arrange in a small number of macroscopically large Bethe strings. The leading order in the large L limit is known to be expressed through a contour integral of a dilogarithm. Here we derive the sub-leading term. Our analysis is based on a new contour-integral representation of the inner product in terms of a Fredholm determinant. We give two derivations of the sub-leading term. Besides a direct derivation by solving a Riemann–Hilbert problem, we give a less rigorous, but more intuitive derivation by field-theoretical methods. For that we represent the Fredholm determinant as an expectation value in a Fock space of chiral fermions and then bosonize. We construct a collective field for the bosonized theory, the short wave-length part of which may be evaluated exactly, while the long wave-length part is amenable to a $1/L$ expansion. Our treatment thus results in a systematic $1/L$ expansion of structure factors within the Sutherland limit.

Closed superstring amplitudes, single-valued multiple zeta values and the Deligne associator

S Stieberger

2014 *J. Phys. A: Math. Theor.* **47** 155401

We revisit the tree-level closed superstring amplitude and identify its α'-expansion as series with single-valued multiple zeta values as coefficients. The latter represent a subclass of multiple zeta values originating from single-valued multiple polylogarithms at unity. Moreover, the α'-expansion of the closed superstring amplitude can be cast into the same algebraic form as the open superstring amplitude: the closed superstring amplitude is essentially the single-valued version of the open superstring amplitude. This fact points to a deeper connection between gauge and gravity amplitudes than is implied by Kawai–Lewellen–Tye relations. Furthermore, we argue that the Deligne associator carries the relevant information on the closed superstring amplitude. In particular, we give an explicit representation of the Deligne associator in terms of Gamma functions modulo squares of commutators of the underlying Lie algebra. This form of the associator can be interpreted as the four-point closed superstring amplitude.

FIELD THEORY AND STRING THEORY

Antonio Pittelli

Secret symmetries of type IIB superstring theory on $\mathrm{AdS}_3 \times S^3 \times M^4$

Antonio Pittelli, Alessandro Torrielli and Martin Wolf

2014 *J. Phys. A: Math. Theor.* **47** 455402

We establish features of so-called Yangian secret symmetries for AdS_3 type IIB superstring backgrounds, thus verifying the persistence of such symmetries to this new instance of the AdS/CFT correspondence. Specifically, we find two *a priori* different classes of secret symmetry generators. One class of generators, anticipated from the previous literature, is more naturally embedded in the algebra governing the integrable scattering problem. The other class of generators is more elusive and somewhat closer in its form to its higher-dimensional AdS_5 counterpart. All of these symmetries respect left-right crossing. In addition, by considering the interplay between left and right representations, we gain a new perspective on the AdS_5 case. We also study the $\mathcal{RTT}$-realisation of the Yangian in AdS_3 backgrounds, thus establishing a new incarnation of the Beisert–de Leeuw construction.

Azat M Gainutdinov

Lattice W-algebras and logarithmic CFTs

A M Gainutdinov, H Saleur and I Yu Tipunin

2014 *J. Phys. A: Math. Theor.* **47** 495401

This paper is part of an effort to gain further understanding of 2D logarithmic conformal field theories (LCFTs) by exploring their lattice regularizations. While all work so far has dealt with the Virasoro algebra (or the product $\mathrm{Vir} \otimes \overline{\mathrm{Vir}}$), the best known (although maybe not the most relevant physically) LCFTs in the continuum are characterized by a W-algebra symmetry, whose presence is powerful, but whose role as a 'symmetry' remains mysterious. We explore here the origin of this symmetry in the underlying lattice models. We consider $U_q\mathfrak{sl}(2)$ XXZ spin chains for q a root of unity, and argue that the centralizer of the 'small' quantum group $\overline{U}_q\mathfrak{sl}(2)$ goes over the W-algebra in the continuum limit. We justify this identification by representation theoretic arguments, and give, in particular, lattice versions of the W-algebra generators. In the case $q = i$, which corresponds to symplectic fermions at central charge $c = -2$, we provide a full analysis of the scaling limit of the lattice Virasoro and W generators, and show in details how the corresponding continuum Virasoro and W-algebras are obtained. Striking similarities between the lattice W algebra and the Onsager algebra are observed in this case.

Fluid and plasma theory

Gregory Falkovich
Section Editor

The fluid and plasma theory section of JPhysA offers a place for researchers working across a wide range of disciplines to share theoretical and mathematical results on both dynamics and statistics of fluid and plasma phenomena.

Theory of fluids involves a very wide scope of mathematical subjects and theoretical methods, from topology, algebra and integrability to modern stochastic methods, as illustrated by the excellent papers in this collection.

The notoriously difficult problem of turbulence slowly reveals its secrets. The phenomenon of explosive formation of an anomalous turbulence spectrum, which was observed in numerical simulations, has now been obtained analytically (Self-similar solution in the Leith model of turbulence: anomalous power law and asymptotic analysis V N Grebenev *et al* 2014 *J. Phys. A: Math. Theor.* **47** 025501).

An important advance has been achieved in the old search (going back to Kelvin) for possible relationships between energy and topology. The influence of internal twist on the minimum magnetic energy levels of knots has been determined by using analytical results for the constrained minimum energy of magnetic knots. (On the groundstate energy spectrum of magnetic knots and links, Renzo L Ricca and Francesca Maggioni, 2014 *J. Phys. A: Math. Theor.* **47** 205501.)

For a magnetized plasma, one is able to build a truncation from kinetics to hydrodynamics which preserves a Hamiltonian structure (Hamiltonian closures for two-moment fluid models derived from drift-kinetic equations, E Tassi 2014 *J. Phys. A: Math. Theor.* **47** 195501).

FLUID AND PLASMA THEORY

Vladimir N Grebenev

Self-similar solution in the Leith model of turbulence: anomalous power law and asymptotic analysis

V N Grebenev, S V Nazarenko, S B Medvedev, I V Schwab and Yu A Chirkunov

2014 *J. Phys. A: Math. Theor.* **47** 025501

We consider a Leith model of turbulence (Leith C 1967 *Phys. Fluids* **10** 1409) in which the energy spectrum obeys a nonlinear diffusion equation. We analytically prove the existence of a self-similar solution with a power-law asymptotic on the low-wavenumber end and a sharp boundary on the high-wavenumber end, which propagates to infinite wavenumbers in a finite-time t_*. We prove that this solution has a power-law asymptotic with an anomalous exponent x^*, which is less than the Kolmogorov value, $x^* > 5/3$. This is a result that was previously discovered by numerical simulations in Connaughton and Nazarenko (2004 *Phys. Rev. Lett.* **92** 044501). We also prove the convergence to this self-similar solution of the spectrum evolving from an arbitrary finitely supported initial data as $t \to t_*$.

Renzo L Ricca

On the groundstate energy spectrum of magnetic knots and links

Renzo L Ricca and Francesca Maggioni

2014 *J. Phys. A: Math. Theor.* **47** 205501

By using analytical results for the constrained minimum energy of magnetic knots we determine the influence of internal twist on the minimum magnetic energy levels of knots and links, and by using ropelength data from the RIDGERUNNER tightening algorithm (Ashton *et al* 2011 *Exp. Math.* **20** 57–90) we obtain the groundstate energy spectra of the first 250 prime knots and 130 prime links. The two spectra are found to follow an almost identical logarithmic law. By assuming that the number of knot types grows exponentially with the topological crossing number, we show that this generic behavior can be justified by a general relationship between ropelength and crossing number, which is in good agreement with former analytical estimates (Buck and Simon 1999 *Topol. Appl.* **91** 245–57, Diao 2003 *J. Knot Theory Ramifications* **12** 1–16). Moreover, by considering the ropelength averaged over a given knot family, we establish a new connection between the averaged ropelength and the topological crossing number of magnetic knots.

FLUID AND PLASMA THEORY

Emanuele Tassi

Hamiltonian closures for two-moment fluid models derived from drift-kinetic equations

E Tassi

2014 *J. Phys. A: Math. Theor.* **47** 195501

We derive the conditions under which the fluid models obtained from the first two moments of Hamiltonian drift-kinetic systems of interest to plasma physics, preserve a Hamiltonian structure. The adopted procedure consists of determining closure relations that allow to truncate the Poisson bracket of the drift-kinetic system, expressed in terms of the moments, in such a way that the resulting operation is a Poisson bracket for functionals of the first two fluid moments. The analysis is carried out for a class of full drift-kinetic equations and also for drift-kinetic systems in which a splitting between an equilibrium distribution function and a perturbation is performed. In the former case we obtain that the only closure that leads to a Poisson bracket, without involving operators or an explicit dependence on the spatial coordinates, corresponds to that of an ideal adiabatic gas made of molecules possessing one degree of freedom. In the latter case, Hamiltonian closures turn out to be those in which the second moment is a linear combination of the first two moments, which can be seen as a linearization of the Hamiltonian closure of the full drift-kinetic case. A number of weakly-3D Hamiltonian-reduced fluid models of interest, for instance for tokamak plasmas, can be derived in this way and, viceversa given a fluid model with a Hamiltonian structure of a certain type, a parent Hamiltonian drift-kinetic model can then be identified. We make use of this correspondence to identify the drift-kinetic models from which Hamiltonian fluid models for magnetic reconnection and compressible plasma dynamics in the presence of a static but inhomogeneous magnetic field can be derived. The Casimir invariants of the Poisson brackets of the derived fluid models are also discussed. It is also shown that the Poisson structure for the fluid model derived from the full drift-kinetic system coincides with that of a reduced fluid model, when using the fluid velocity instead of the momentum as a dynamical variable.

Topical reviews

Time scale of diffusion in molecular and cellular biology

D Holcman and Z Schuss

2014 *J. Phys. A: Math. Theor.* **47** 173001

Single- and coupled-channel radial inverse scattering with supersymmetric transformations

Daniel Baye, Jean-Marc Sparenberg, Andrey M Pupasov-Maksimov and Boris F Samsonov

2014 *J. Phys. A: Math. Theor.* **47** 243001

Generalized probability theories: what determines the structure of quantum theory?

Peter Janotta and Haye Hinrichsen

2014 *J. Phys. A: Math. Theor.* **47** 323001

Revealing networks from dynamics: an introduction

Marc Timme and Jose Casadiego

2014 *J. Phys. A: Math. Theor.* **47** 343001

Mathematical and physical aspects of complex symmetric operators

Stephan Ramon Garcia, Emil Prodan and Mihai Putinar

2014 *J. Phys. A: Math. Theor.* **47** 353001

Entanglement typicality

Oscar C O Dahlsten, Cosmo Lupo, Stefano Mancini and Alessio Serafini

2014 *J. Phys. A: Math. Theor.* **47** 363001

Integrability of the $AdS_5 \times S^5$ superstring and its deformations

Stijn J van Tongeren

2014 *J. Phys. A: Math. Theor.* **47** 433001

A smooth introduction to the wavefront set

Christian Brouder, Nguyen Viet Dang and Frédéric Hélein

2014 *J. Phys. A: Math. Theor.* **47** 443001

Entanglement witnesses: construction, analysis and classification

Dariusz Chruściński and Gniewomir Sarbicki

2014 *J. Phys. A: Math. Theor.* **47** 483001

Nonlinear lattice waves in heterogeneous media

T V Laptyeva, M V Ivanchenko and S Flach

2014 *J. Phys. A: Math. Theor.* **47** 493001

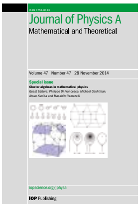
2014 special issues

Fifty years of Bell's theorem

Guest Editors: Nicolas Brunner, Otfried Gühne and Marcus Huber

This special issue of *Journal of Physics A: Mathematical and Theoretical* presents recent results on Bell inequalities and entanglement theory, but also topical reviews and personal views on John Bell and his theorem. Includes the following review articles:

- The two Bell's theorems of John Bell
  H M Wiseman 2014 *J. Phys. A: Math. Theor.* **47** 424001
- Quantifying entanglement resources
  Christopher Eltschka and Jens Siewert 2014 *J. Phys. A: Math. Theor.* **47** 424005
- Quantum metrology from a quantum information science perspective
  Géza Tóth and Iagoba Apellaniz 2014 *J. Phys. A: Math. Theor.* **47** 424006



Cluster algebras in mathematical physics

Guest Editors: Philippe Di Francesco, Michael Gekhtman, Atsuo Kuniba and Masahito Yamazaki

This special issue of *Journal of Physics A: Mathematical and Theoretical* contains reviews and original research articles on cluster algebras and their applications to mathematical physics. Featuring the following review articles:

- Y-systems, Q-systems, and 4D $N = 2$ supersymmetric QFT
  Sergio Cecotti and Michele Del Zotto 2014 *J. Phys. A: Math. Theor.* **47** 474001
- Periodic cluster mutations and related integrable maps
  Allan P Fordy 2014 *J. Phys. A: Math. Theor.* **47** 474003
- Bipartite field theories, cluster algebras and the Grassmannian
  Sebastián Franco, Daniele Galloni and Alberto Mariotti 2014 *J. Phys. A: Math. Theor.* **47** 474004

iopscience.org/jphysa/specialissues

Forthcoming special issues

Look out for our forthcoming special issues throughout the year.

Exactly Solved Models and Beyond: a special issue in honour of R J Baxter's 75th birthday

Guest Editors: Murray Batchelor, Vladimir Bazhanov and Vladimir Mangazeev

Localisation techniques in quantum field theories

Guest Editors: Vasily Pestun and Maxim Zabzine

Mathematics for the fluid Earth

Guest Editors: Tobias Kuna and Sandro Vaienti

Modelling and inference in the dynamics of complex interaction networks: advanced approximation techniques

Guest Editors: Manfred Opper, Yasser Roudi and Peter Sollich

Best Paper Prize

Since 2009, *Journal of Physics A: Mathematical and Theoretical* has awarded a Best Paper Prize, which serves to celebrate well-written papers that make a significant contribution to their field.

The Editorial Board awarded the 2014 Best Paper Prize to the following papers, which were considered to excel in the criteria of novelty, achievement, potential impact and presentation.

- Robust self-testing of the singlet
  M McKague, T H Yang and V Scarani
  2012 *J. Phys. A: Math. Theor.* **45** 455304

- Holography, unfolding and higher spin theory
  M A Vasiliev
  2013 *J. Phys. A: Math. Theor.* **46** 214013

You can read these papers and see the previous Best Paper Prize winners at **iopscience.org/jphysa/bestpaperprize15**.

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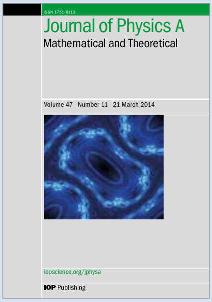

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