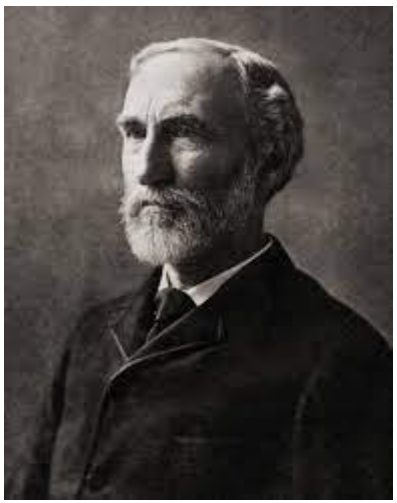

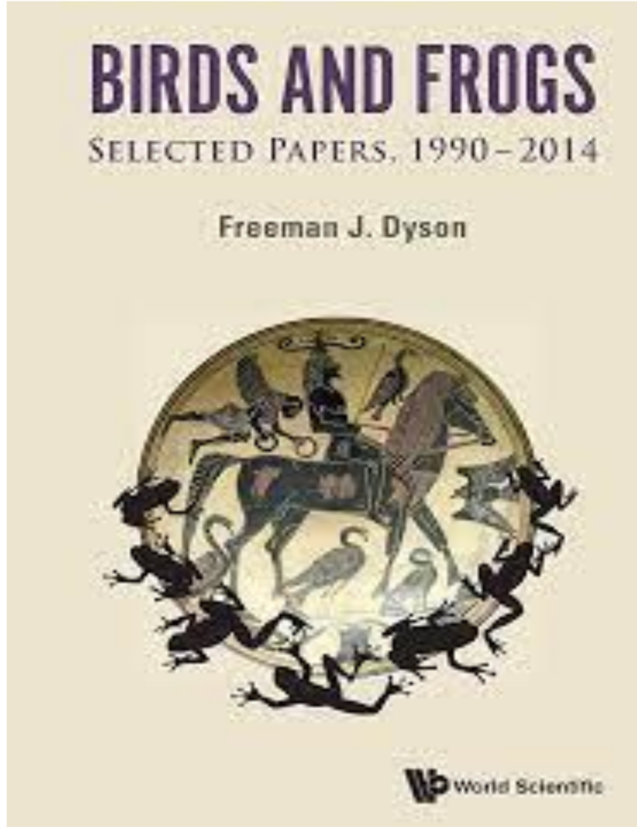

MeccanicaMente

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AB Streamlined CV

-Formazione:

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Dottorato in Applied Mathematics (2008), King's College London

-ASN

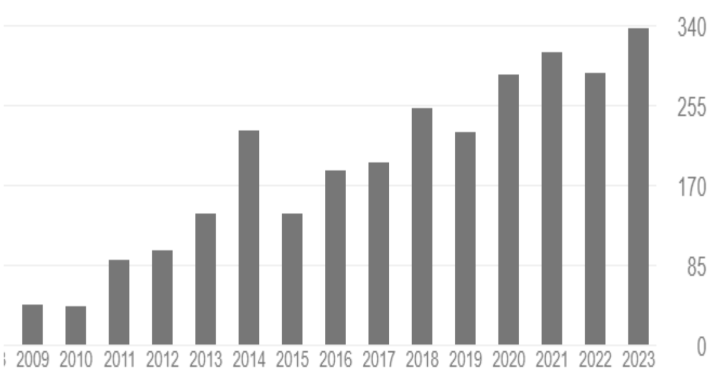
Abilitato Professore di I Fascia in MAT/07 e FIS/07
Abilitato Professore di II Fascia in MAT/07, FIS/07, FIS/02a2 FIS/02b2

-Afferenze: INdAM (GNFM), UMI (Giunta AI), INFN, ARxIV, CMCC, Sant'Anna, PhD-AI

-Mansioni Editoriali: Neural Networks (Elsevier), Nature Sci. Rep., Proc. Royal Society

-Grants in essere: MAECI (Italy-Israel), PRIN (Mat07), PNRR (Foundation AI) [←expected…]

-Publicazioni : >100 in Meccanica Statistica ad ampio spettro



Problemi diretti:
- -Neural Networks and Learning Machines
- -Spin Glasses and Random Networks
- -Reaction Kinetics & Biological Computing
- -Mathematical models & methods

Problemi inversi:
- -Stochastic Processes in Quantitative Biology (mainly immune response)
- -HD Statistical Inference in Quantitative Biology (mainly cancerogenesis)
- -Machine Learning per Quantitative Biology (genomics & trajectory reconstruction)

Structure of the talk

We use a simple physical model to inspect the techniques (mean field Ising model, Curie-Weiss)

1) Physicist approach to ferromagnetic phase transitions

2) Statistical mechanical description (probabilistic methods)

3) Analytical mechanical description (PDE techniques)

From mean-field Ising to mean field neural networks:

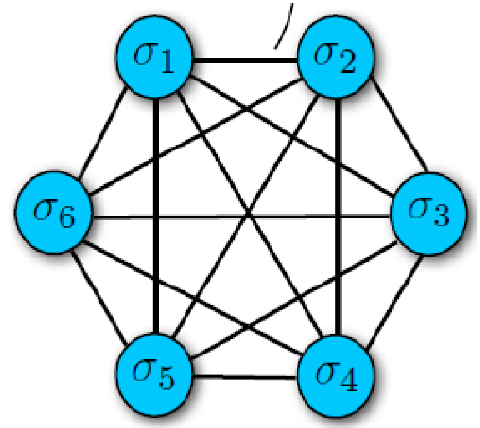
- Hopfield model for pattern recognition
- Hopfield model for machine learning

1 Mean field phase transitions: heuristics

Oscillatori di Kuramoto: video
Spin di Curie-Weiss: video

We use the *Free Energy F*

$$F = E - TS \tag{1}$$



$$E = H(\sigma|J,h) = -\frac{1}{N}\sum_{i<j}^{N,N} J_{ij}\sigma_i\sigma_j - h\sum_i^N \sigma_i = -N\left(J\frac{m^2}{2} + hm\right) \tag{2}$$

and, with the choice $J_{ij} = J > 0$, in the large $N \to \infty$ limit, we write

$$m = \frac{1}{N}\sum_i^N \sigma_i$$

$$\mathscr{P}(\sigma_1, \sigma_2, ..., \sigma_N) = \prod_{i=1}^N \mathscr{P}(\sigma_i), \tag{3}$$

$$\mathscr{P}(\sigma_i) = \left(\frac{1+M}{2}\right)\delta(\sigma_i - 1) + \left(\frac{1-M}{2}\right)\delta(\sigma_i + 1), \tag{4}$$

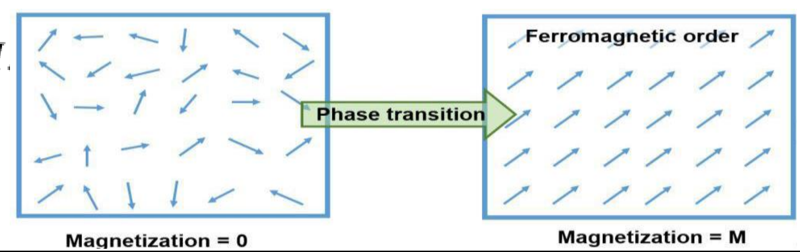
$$\langle\sigma\rangle = \left[\left(\frac{1+M}{2}\right)\cdot(+1) + \left(\frac{1-M}{2}\right)\cdot(-1)\right] = M, \quad \longleftarrow \quad \lim_{N\to\infty}\mathscr{P}(m) = \delta(m - M)$$

$$S(\sigma) = -\sum_{\sigma_i=\pm 1}\mathscr{P}(\sigma_i)\ln\mathscr{P}(\sigma_i) = -N\left[\left(\frac{1+M}{2}\right)\ln\left(\frac{1+M}{2}\right) + \left(\frac{1-M}{2}\right)\ln\left(\frac{1-M}{2}\right)\right]$$

- Control parameter: T, h, J.
- Order parameter: $m = \frac{1}{N}\sum_i^N \sigma_i$, whose mean is $\langle m\rangle = M$.

furthermore, keep in mind that the temperature obeys a random walk (i.e. Fourier PDE).

T alta (vince Smax) **T bassa (vince Emin)**



thus we can write $F = F(M|T,h,J)$ as

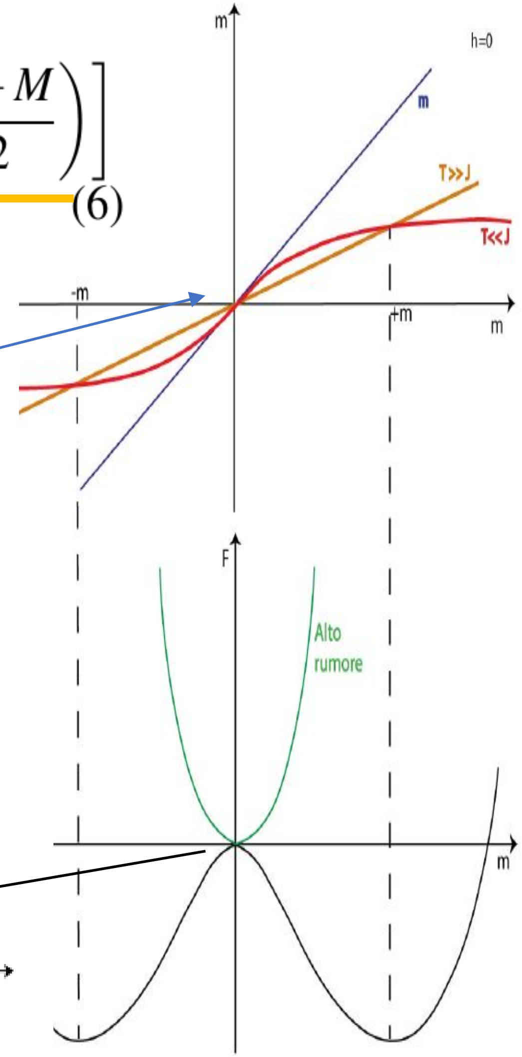
$$F = -N\left[\left((\frac{J}{2}M^2 + hM\right) - T\left(\frac{1+M}{2}\right)\ln\left(\frac{1+M}{2}\right) + \left(\frac{1-M}{2}\right)\ln\left(\frac{1-M}{2}\right)\right] \quad (6)$$

and impose Thermodynamics by requiring (remember $F = E - TS$)

$$\frac{dF}{dM} = 0 \ \ \& \ \ \frac{d^2F}{dM^2} \geq 0 \quad (7)$$

$$\frac{dF}{dM} = 0 \quad \rightarrow \quad M = \tanh\left(\frac{JM+h}{T}\right) \quad (8)$$

$$\frac{d^2F}{dM^2} = \chi \quad \rightarrow \quad \chi = \frac{C}{T-T_c} \quad (9)$$



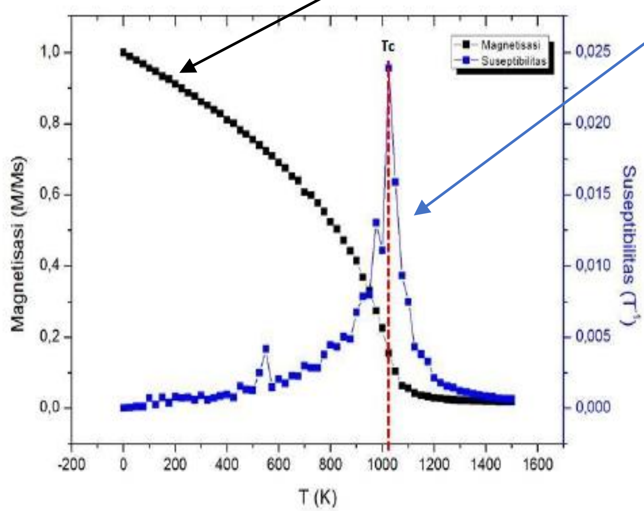
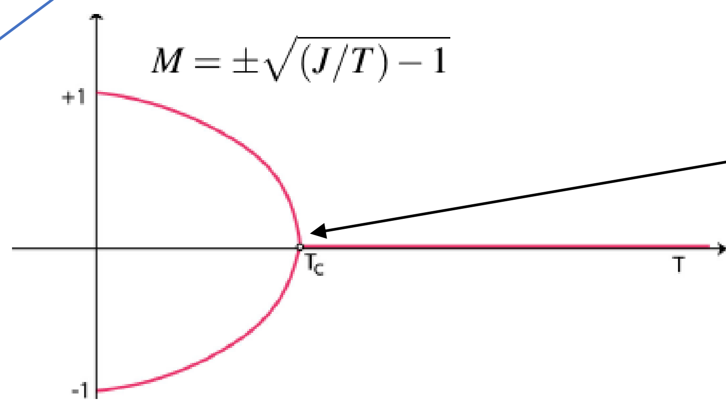


Figure 3. Graph of magnetization value [13]

$$M = \pm\sqrt{(J/T)-1}$$



We expand for small M at $h = 0$ to see spontaneously broken symmetries

Memo: $\tanh(x) \sim x - x^3/3 + O(x^5)$

$$M = \tanh(\frac{J}{T}M) \sim \frac{J}{T}M - (\frac{J}{T})^3\frac{M^3}{3} \rightarrow M(\frac{J}{T}-1) \sim CM^3 \rightarrow M \propto \pm\sqrt{\frac{J}{T}-1}$$

Shannon Entropy = Boltzmann entropy?

$$\begin{aligned} S[\sigma] &= \ln\Omega, \quad \Omega = \binom{N}{N_A} = \binom{N}{N_B} & (10)\\ &= \ln\left(\frac{N!}{N_A!N_B!}\right) = \ln N! - \ln N_A! - \ln N_B! & (11)\\ &\sim N\ln N - N_A \ln N_A - N_B \ln N_B - (N - N_A - N_B) = & (12)\\ &= (N_A + N_B)\ln N - N_A \ln N_A - N_B \ln N_B & (13)\\ &= -N_A \ln\left(\frac{N_A}{N}\right) - N_B \ln\left(\frac{N_B}{N}\right) = -N_A \ln \rho_A - N_B \ln \rho_B & (14) \end{aligned}$$

by which, calling $\rho_A = N_A/N$ and $\rho_B = N_B/N$, the intensive entropy reads

$$\begin{aligned} \frac{S[\sigma]}{N} &= -\rho_A \ln \rho_A - \rho_B \ln \rho_B = \rho \ln \rho + (1-\rho)\ln(1-\rho), & (15)\\ \frac{S[\sigma]}{N} &= -(\frac{1+M}{2})\ln(\frac{1+M}{2}) - (\frac{1-M}{2})\ln(\frac{1-M}{2}). & (16) \end{aligned}$$

When the Entropy is extremized? $\rho = 1/2 \rightarrow M = 0$.

whose extremization (as expected) returns

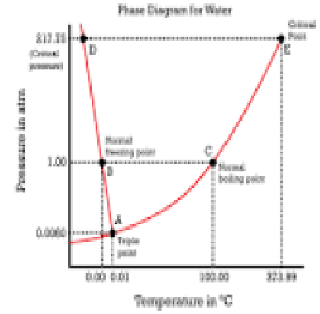
$$\frac{d}{d\rho}\frac{S[\sigma]}{N} = 0 \rightarrow -\ln\rho - \frac{\rho}{\rho} + \ln(1-\rho) + \frac{(1-\rho)}{(1-\rho)} = -\ln\left(\frac{\rho}{1-\rho}\right) \rightarrow \rho = 1/2.$$

namely $M = 0$.

Statistical Mechanics in a nutshell

Statistical mechanics has its genesis in Physics (late 800/early 900)

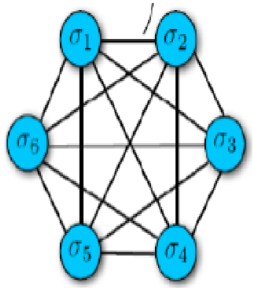
- It was born as an alternative route (w.r.t system's of ODE) to capture thermodynamic properties of large ensembles of (interacting) units
- Grounded on probabilistic techniques reveals «emerging capabilities»
- Focuses on few effective degreees of freedom (control and order parameters)



alla ricerca delle proprietà emergenti….

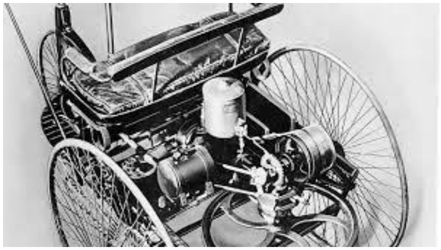
It has then «exported» in Neural Networks (1985/2000, AGS-Theory)

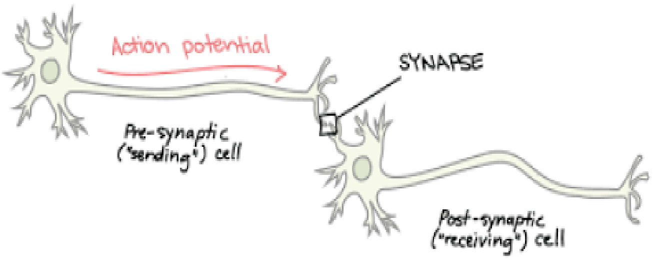
- Basic Reference: discrete statistical mechanics (spin systems) [two-state neurons]
- Simple models: paramagnetic → ferromagentic transitions (Curie-Weiss)
- Complex models: spin glasses & Parisi Theory [synapses are both positive and negative]



(almost-deterministic) PROTOCOL:

1) Define the cost-function (i.e. the Hamiltonian in Physics).
2) Define control and order parameters
3) Introduce the free energy
4) Extremize the free energy F/A w.r.t. the order parameters to work out phase diagrams in the control parameters.





FREE ENERGY = ENERGY MINUS TEMPERATURE TIMES ENTROPY (not so different from ML…)

meccanicamente….

2 F in Statistical Mechanics: Mom. Gen. Func.

$$E = H(\sigma|J,h) = -\frac{J}{2N}\sum_{i,j}^{N,N}\sigma_i\sigma_j - h\sum_i^N\sigma_i = -N\left(J\frac{m^2}{2} + hm\right) \tag{17}$$

$$\mathscr{P}(\{\sigma\}) = \frac{1}{Z(T,J,h)}e^{-\frac{H(\sigma|J,h)}{T}} = \frac{e^{\frac{\frac{J}{2N}\sum_{i,j}^{N,N}\sigma_i\sigma_j + h\sum_i^N\sigma_i}{T}}}{\sum_\sigma^{2^N} e^{\frac{J}{2N}\sum_{i,j}^{N,N}\sigma_i\sigma_j + h\sum_i^N\sigma_i}}$$

$$\begin{aligned} F(m|T,J,h) &= -\frac{T}{N}\ln\sum_\sigma^{2^N}\exp\left(-\frac{H(\sigma|J,h)}{T}\right) \\ &= -\frac{T}{N}\ln\sum_\sigma^{2^N}\exp\left(\frac{\frac{JN}{2}m^2 + hNm}{T}\right) \end{aligned} \tag{18}$$

- First moment: magnetization $\frac{dF}{dh} = -\langle m\rangle$
- Second moment: susceptibility $\frac{d^2F}{dh^2} = \chi = N\left(\langle m^2\rangle - \langle m\rangle^2\right)$

$$\langle m\rangle = \frac{\sum_\sigma^{2^N} m\, e^{-H/T}}{\sum_\sigma^{2^N} e^{-H/T}} \qquad m = \frac{1}{N}\sum_i^N \sigma_i$$

3 Symmetry breaking: Guerra's interpolation

$\lim_{N\to\infty} \mathscr{P}(m) = \delta(m-M)$ cum grano salis...

Interpolation

$$F(t) = -\frac{T}{N}\ln\sum_{\sigma}^{2^N}\exp\left[(t)\left(\frac{J}{TN}\sum_{i<j}^{N,N}\sigma_i\sigma_j\right)+(1-t)\left(\frac{JM}{T}\sum_{i=1}^{N}\sigma_i\right)+h\sum_{i=1}^{N}\sigma_i\right] \tag{13}$$

Solution via Fundamental Theorem of Calculus

$$F(T,J,h) = F(t=1) = F(t=0) + \int_0^1 \dot{F}(t')dt' \tag{14}$$

Thus we are left to two calculations to do, the *one-body* (i.e. $F(t=0)$) and the *streaming* (i.e. $d_tF(t)$).

Let us start with the former, the one-body term:

$$F(t=0) = -\frac{T}{N}\ln\sum_{\sigma}^{2^N}\exp\left[\left(\frac{JM}{T}\sum_{i=1}^{N}\sigma_i\right)+h\sum_{i=1}^{N}\sigma_i\right] \tag{15}$$

$$= -\frac{T}{N}\ln\sum_{\sigma}^{2^N}\exp\left[\left(\frac{JM+h}{T}\right)\sum_{i=1}^{N}\sigma_i\right] \tag{16}$$

$$= -\frac{T}{N}\ln\sum_{\sigma}^{2^N}\prod_{i=1}^{N}\exp\left[\left(\frac{JM+h}{T}\right)\sigma_i\right] \tag{17}$$

$$= -T\ln\left[2\cosh\left(\frac{JM+h}{T}\right)\right] \tag{18}$$

Let us focus now on the streaming, namely the evaluation of $\int_0^1 \dot{F}(\mathbf{t})dt$. Before proceeding note that $m = \frac{1}{N}\sum_i \sigma_i$ thus

- where it is simple to evaluate (e.g. in the high T limit) $\frac{1}{\sqrt{N}}\sum_i^N \sigma_i \sim Z[0,1]$
- thus $\lim_{N\to\infty} \mathscr{P}(m) = \delta(m - M)$ $\qquad m = \frac{1}{N}\sum_i^N \sigma_i$
- or equivalently $\lim_{N\to\infty}\langle (m-M)^2\rangle = 0$ that means $\langle m^2\rangle - 2M\langle m\rangle = -M^2$

Let us start now with the derivative of the interpolating free energy, that reads

$$F(t) = -\frac{T}{N}\ln\sum_{\sigma}^{2^N}\exp\left[(t)\left(\frac{J}{TN}\sum_{i<j}^{N,N}\sigma_i\sigma_j\right) + (1-t)\left(\frac{JM}{T}\sum_{i=1}^{N}\sigma_i\right) + h\sum_{i=1}^{N}\sigma_i\right] \tag{19}$$

hence its t-derivative reads (in the thermodynamic limit $N \to \infty$)

$$\frac{dF}{dt} = -\frac{J}{2}\left(\langle m^2\rangle - 2M\langle m\rangle\right) = \frac{JM^2}{2} \tag{20}$$

that does no longer depends on t and, thus, its integral in $t \in (0,1)$ is just multiplication by one.

Pasting the two pieces together, we can write

$$F(m|T,J,h) = F(t=1) = F(t=0) + \int_0^1 \dot{F}(t')dt' \quad (21)$$

$$= -T\ln\left[2\cosh\left(\frac{JM+h}{T}\right)\right] + \frac{JM^2}{2} \quad (22)$$

Thus we can finally extremize the latter to obtain

$$\frac{dF}{dM} = 0 \quad \to \quad \boxed{\tanh\left(\frac{JM+h}{T}\right) = M.} \quad (23)$$

What can we say on the Susceptiblity? Gaussian Fluctuation Theory

Memo:

$$Z = \sum_{\sigma}^{2^N} \exp\left(t(\frac{Nm^2}{2T}) + (1-t)\frac{NMm,}{T}\right)$$

- We first rescale the magnetization properly, i.e. $\tilde{m} = \sqrt{N}(m-M)$
- We then prove that $\frac{d}{dt}\langle f(\sigma)\rangle = \frac{1}{2T}\left(\langle f(\sigma)\tilde{m}^2\rangle - \langle f(\sigma)\rangle\langle\tilde{m}^2\rangle\right)$
- use it with the choice $f(\sigma) = \tilde{m}^2 \to \frac{d}{dt}\langle\tilde{m}^2\rangle = \frac{1}{2T}\langle\tilde{m}^2\rangle^2$
- the solution of the ODE is

$$\boxed{\langle\tilde{m}^2\rangle = \frac{C}{1-t\frac{J}{T}} \to_{t=1} \frac{C'}{T-J}}$$

4 Symmetry breaking as bifurcation: PDE approach

It is useful to introduce $A(T,J,h) = -\frac{F(T,J,h)}{T}$ such that

$$A(T,J,h) = \frac{1}{N}\ln\sum_{\sigma}\exp\left(\frac{J}{T}\frac{N}{2}m^2 + \frac{h}{T}Nm\right) \tag{24}$$

and pave the *Mechanical Analogy* (note: these are all Joules by Legendre)

$$A(t,\ x) = \frac{1}{N}\ln\sum_{\sigma}\exp\left(-t\frac{N}{2}m^2 + xNm\right) \tag{25}$$

such that it is trivial to check that

$$\frac{\partial A}{\partial t} = = \frac{1}{N}\frac{1}{Z}\sum_{\sigma}^{2^N}\left(-\frac{1}{2}Nm^2\right)e^{-H/T} = -\frac{1}{2}\langle m^2\rangle, \tag{26}$$

$$\frac{\partial A}{\partial x} = \frac{1}{N}\frac{1}{Z}\sum_{\sigma}^{2^N}(m)\,e^{-H/T} = \langle m\rangle. \tag{27}$$

As a consequence, if I introduce a potential $V_N(t,x) = \frac{1}{2}\left(\langle m\rangle^2 - \langle m^2\rangle\right)$ by construction the following HJ PDE holds (even at finite volumes)

$$\frac{\partial A}{\partial t} + \frac{1}{2}\left(\frac{\partial A}{\partial x}\right)^2 + V_N(t,x) = 0. \tag{28}$$

$$A(t,\ x) = \frac{1}{N}\ln\sum_{\sigma}\exp\left(-t\frac{N}{2}m^2 + xNm\right) \tag{23}$$

$$\frac{\partial A}{\partial t} + \frac{1}{2}\left(\frac{\partial A}{\partial x}\right)^2 + V_N(t,x) = 0. \tag{26}$$

Now, as $\lim_{N\to\infty} V_N(t,x) = 0$, in the thermodynamic limit we are left with a Galilean motion, where the *velocity* of the unitary mass is $\langle m\rangle$ and the related trajectory reads as $x(t) = x_0 + \langle m\rangle t$.

The solution of the HJ PDE is the Cauchy condition $A(t=0, x=x_0)$ plus the integral of the Lagrangian $\mathscr{L}$ over time (and the Lagrangian is just the kinetic term), namely

$$\boxed{A(t,x) = A(t=0, x=x_0) + \int_0^t \mathscr{L}\,dt} \tag{27}$$

We have thus to perform these two evaluations, that read the former

$$\begin{aligned} A(t=0, x=x_0) &= \frac{1}{N}\ln\sum_{\sigma}\exp(x_0 Nm) = \frac{1}{N}\ln\sum_{\sigma}\exp\left(x_0\sum_{i}^{N}\sigma_i\right) && (28) \\ &= \ln 2 + \ln\cosh(x_0) = \ln 2 + \ln\cosh\left(x(t) - \langle m\rangle t\right) && (29) \end{aligned}$$

the latter

$$\int_0^t \mathscr{L} dt = \frac{1}{2}\langle m^2\rangle t \tag{30}$$

Merging the two together, we can write

$$A(t,x) = \ln 2 + \ln\cosh\left(x(t) - \langle m\rangle t\right) + \frac{1}{2}\langle m^2\rangle t, \tag{31}$$

$$A(t = -\frac{J}{T}, x = \frac{h}{T}) = \ln 2 + \ln\cosh\left(\frac{h}{T} + \frac{J}{T}\langle m\rangle\right) - \frac{J}{2T}\langle m^2\rangle \tag{32}$$

namely

$$A(T,J,h) = \ln 2 + \ln\cosh\left(\frac{h+JM}{T}\right) - \frac{J}{2T}M^2, \tag{33}$$

$$d_M A = 0 \rightarrow \boxed{M = \tanh\left(\frac{h+JM}{T}\right).} \tag{34}$$

…è il principio di Minima Azione nel formalismo della Meccanica Analitica!

If we work already in the TD limit we can write a Riemann-Hopf PDE for the magnetization

$$\frac{\partial A}{\partial x} = \frac{1}{N}\frac{1}{Z}\sum_{\sigma}^{2^N}(m)\,e^{-H/T} = \langle m \rangle.$$

$$\frac{\partial}{\partial x}\left(\frac{\partial A}{\partial t} + \frac{1}{2}\left(\frac{\partial A}{\partial x}\right)^2\right) = 0 \rightarrow \boxed{\frac{\partial \langle m \rangle}{\partial t} + \langle m \rangle \frac{\partial \langle m \rangle}{\partial x} = 0} \qquad (37)$$

that is the harmonic oscillator of a Hopf (or Hopf-Lax) bifurcation.
Indeed we can inspect mass conservation to see that

$$\rho(x)dx = \rho(y)dy$$

and, by the Galilean equation of motion we can write (at $h = 0$)

$$x = y + \langle m \rangle t = y + \tanh(\frac{J}{T}\langle m \rangle)t$$

such that the Jacobian reads

$$\frac{dx}{dy} = 1 + \partial_y \langle m \rangle t = 1 + (1 - \tanh^2(\frac{J}{T}\langle m \rangle)t$$

thus for the mass density at a generic point x we get, from the high temperature (where $\langle m \rangle = 0$)

$$\rho(x) = \rho(y)\frac{1}{1 + (1 - \tanh^2(\frac{J}{T}\langle m \rangle)t} = \rho(y)\frac{1}{1+t} = \rho(y)\frac{1}{1 - J/T}$$

namely there is a pole at $T_c = J$, where the spin-flip symmetry becomes spontaneously broken (and, accordingly, a shock happens in this dual representation).

4 Relativity beyond Statistical Reductionism

$$A(t, x) = \frac{1}{N} \ln \sum_{\sigma} \exp\left(-t\frac{N}{2}m^2 + xNm\right) \tag{38}$$

There are two points in the previous PDE approach that *smell relativistic* namely

- the velocity is the magnetization, that is bounded by 1 (and in Classical Mechanics velocity is not bounded).
- The metric tensor is Lorenzian rather than Euclidean (i.e. $-t, +x$ rather than $+t, +x$)

the whole suggests to move from $E = -\frac{1}{2}m^2$ to $E = \sqrt{1+m^2} = 1 - \frac{1}{2}m^2 + O(m^4)$.

- This is the analogous of considering solely binary collisions when using particles
- This is deeply related to the fact that if forces are linear, then energies are quadratic forms

L'**energia totale relativistica** E, nella teoria della relatività ristretta, è data dall'*Energia a riposo* $E_0 = mc^2$ a cui va aggiunta l'*Energia cinetica relativistica* $K = (\gamma - 1)\, mc^2$:

$$E = E_0 + K = mc^2 + (\gamma - 1)\, mc^2 = \gamma\, m\, c^2 = \sqrt{p^2c^2 + m^2c^4}$$

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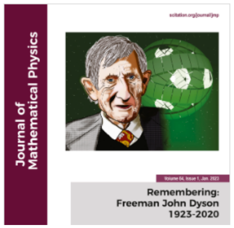

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PAPER

Replica symmetry breaking in neural networks: a few steps toward rigorous results

Elena Agliari[6,1,2], Linda Albanese[3], Adriano Barra[2,3,4] and Gabriele Ottaviani[5]

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Neural Networks
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Generalized Guerra's interpolation schemes for dense associative neural networks

Elena Agliari [a], Francesco Alemanno [b c], Adriano Barra [b d], Alberto Fachechi [b d]

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Quantifying heterogeneity to drug response in cancer-stroma kinetics

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ACS APPLIED MATERIALS & INTERFACES
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Fully Automated Computational Approach for Precisely Measuring Organelle Acidification with Optical pH Sensors

Anil Chandra, Saumya Prasad, Francesco Alemanno, Maria De Luca, Riccardo Rizzo, Roberta Romano, Giuseppe Gigli, Cecilia Bucci, Adriano Barra,* and Loretta L. del Mercato*

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Probing Single-Cell Fermentation Fluxes and Exchange Networks via pH-Sensing Hybrid Nanofibers

Valentina Onesto, Stefania Forciniti, Francesco Alemanno, Krishnadev Narayanankutty, Anil Chandra, Saumya Prasad, Amalia Azzariti, Giuseppe Gigli, Adriano Barra, Andrea De Martino, Daniele De Martino,* and Loretta L. del Mercato*

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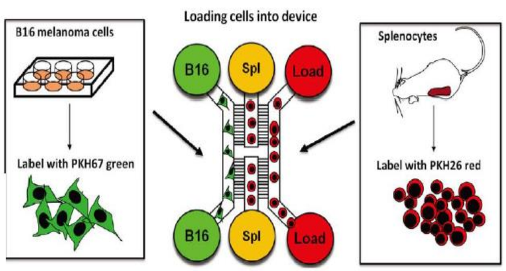
SCIENTIFIC REPORTS
OPEN
SUBJECT AREAS: COMPUTATIONAL BIOPHYSICS

Cancer-driven dynamics of immune cells in a microfluidic environment

Elena Agliari[1], Elena Biselli[1], Adele De Ninno[2], Giovanna Schiavoni[3], Lucia Gabriele[3], Anna Gerardino[4], Fabrizio Mattei[3], Adriano Barra[1] & Luca Businaro[4]



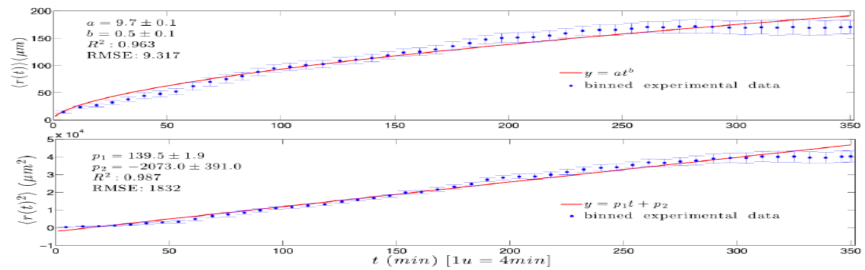
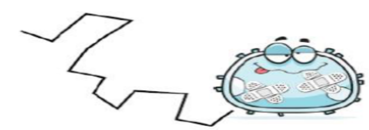
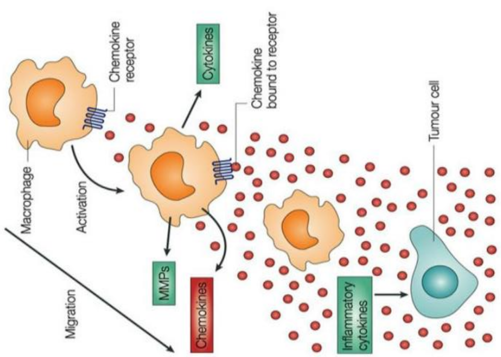


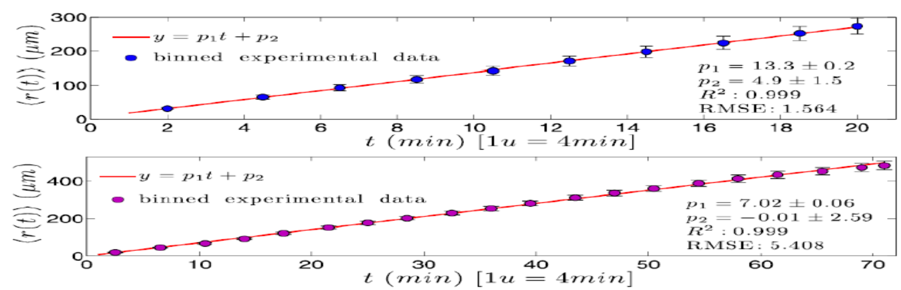
Figure 9 | Mean displacement $\langle r(t) \rangle$ (upper panel) and mean squared displacement $\langle r^2(t) \rangle$ (lower panel) for KO splenocytes. Experimental (binned) data (•) with standard errors are compared with best fits (solid line) whose coefficients are properly shown.



KO leukocytes perform standard diffusion.



OK (WT) leukocytes perform ballistic motion: they perceive the chemoattractant gradient.



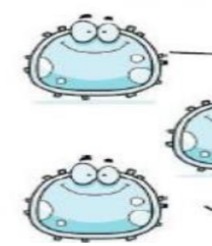
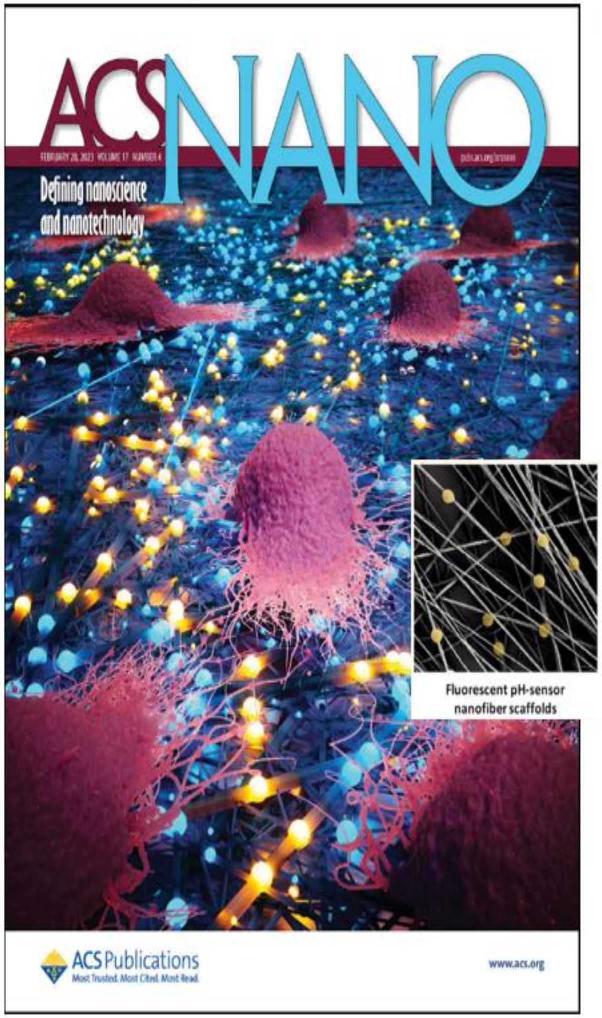
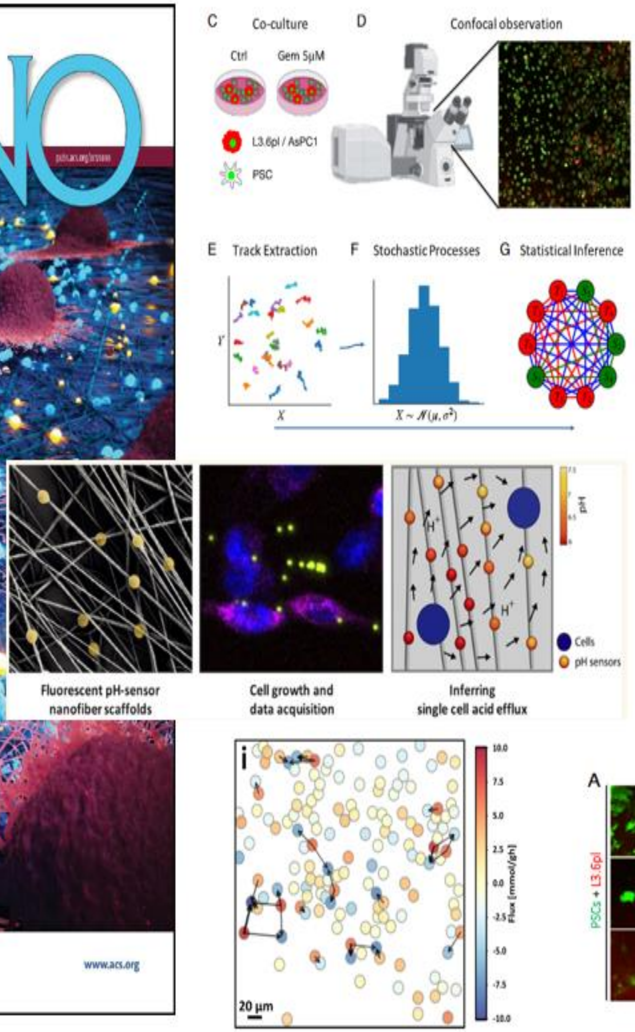
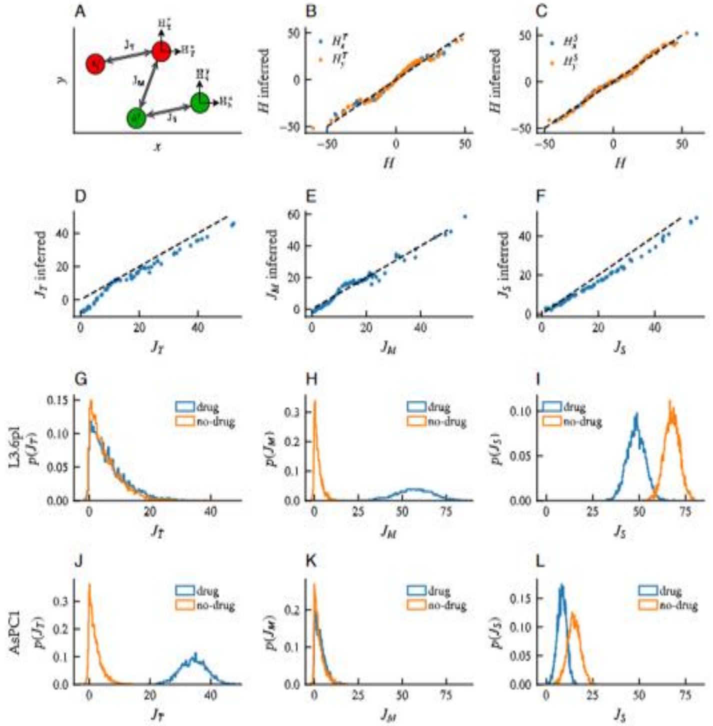


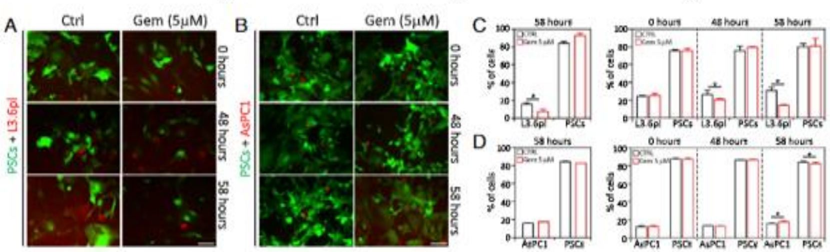
Figure 16 | $\langle r(t) \rangle$ versus t for WT-PRE splenocytes (upper panel) and for WT-POST splenocytes (lower panel). As expected, the mean displacem[ent] grows linearly with time. Binned data (•) with standard errors are compared with best fit (solid line), whose coefficients are also reported.

Theoretical Division [TD] (Dipartimento di Matematica e Fisica, Università del Salento)









une petite parenthèse historique: is Boltzmann = Shannon for these systems?

Vedremo che partendo dalle definizione di entropia data da Boltzmann arriviamo a quella di Shannon.

Il numero ci configurazioni possibili è $\Omega = \dfrac{N!}{N_A! N_B!}$ da cui possiamo definire l'entropia come

$$S \sim \ln \Omega = \ln N! - \ln N_A! - \ln N_B!$$

Per $N \to \infty$ usiamo Stirling ed otteniamo

$$S \sim N \ln N - N_A \ln N_A - N_B \ln N_B \ + (-N + N_A + N_B) \ .$$

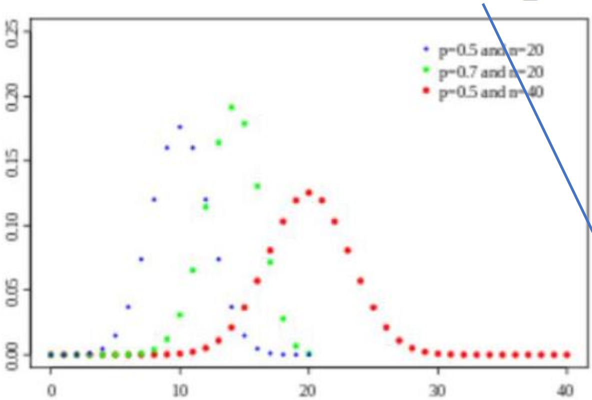
A questo punto aggiungiamo la quantità nulla $[(N_A + N_B) \ln N - N \ln N]$. Quindi possiamo riscrive

$$S \sim N_A \ln \rho_A - N_B \ln \rho_B$$

dove abbiamo introdotto le densità $\rho_A = N_A / N$ e $\rho_B = N_B / N$. Adesso dividiamo tutto per N (per avere l'espressione intensiva) e troviamo

$$s = \frac{S}{N} = -\rho_A \ln \rho_A - \rho_B \ln \rho_B$$

E quindi troviamo che $\rho_{eq} = \dfrac{1}{2}$.



i.e. M=0

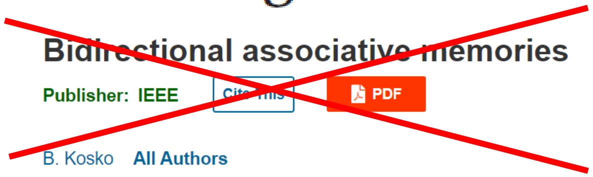
Un altro concetto importante:

QUANTITA' INTENSIVE VS QUANTITA' ESTENSIVE

$$\mu = \sum_{x=0}^{n} x P_{\text{bin}}(x) = np$$

$$\sigma = \sqrt{np(1-p)}$$

From Pavlov Conditioning to Hebb Learning



What do we need?

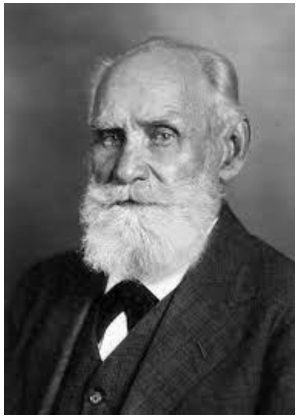
1) A reference model in glassy statistical mechanics:
Switch from serial processing (standard AGS theory) to parallel processing (modern theory)

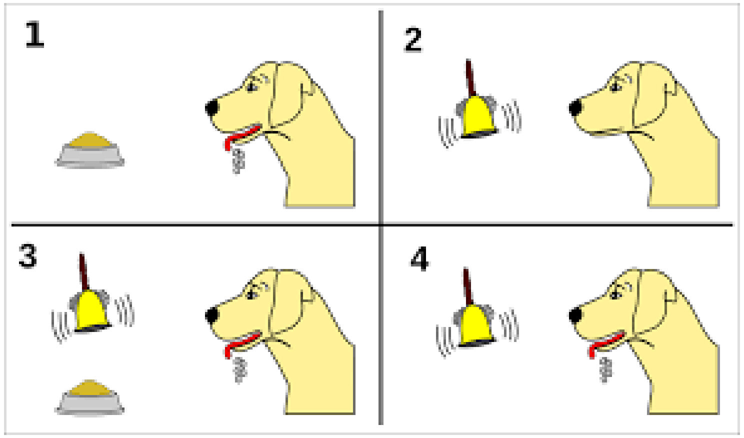
Why standard Hopfiel model can not be the correct reference?

1) A toy model to show that «*neurons that fire together wire together*» (1-bit prescription)

Guerra scheme for 1-bit Classical Conditioning

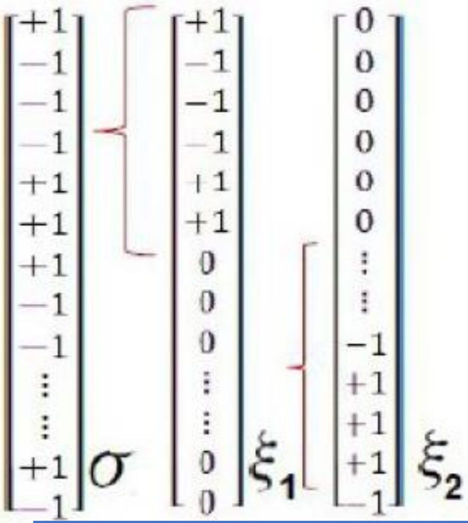
1) The generalization from bits to informative patterns (e.g. ringing bell and food for dogs)





2) A proof that such a generalization converges to the Hebbian kernel

Multi-tasking associative networks: patterns with diluted entries



$$\langle H \rangle = E = -\sum_{\mu}^{p} m_\mu^2,$$

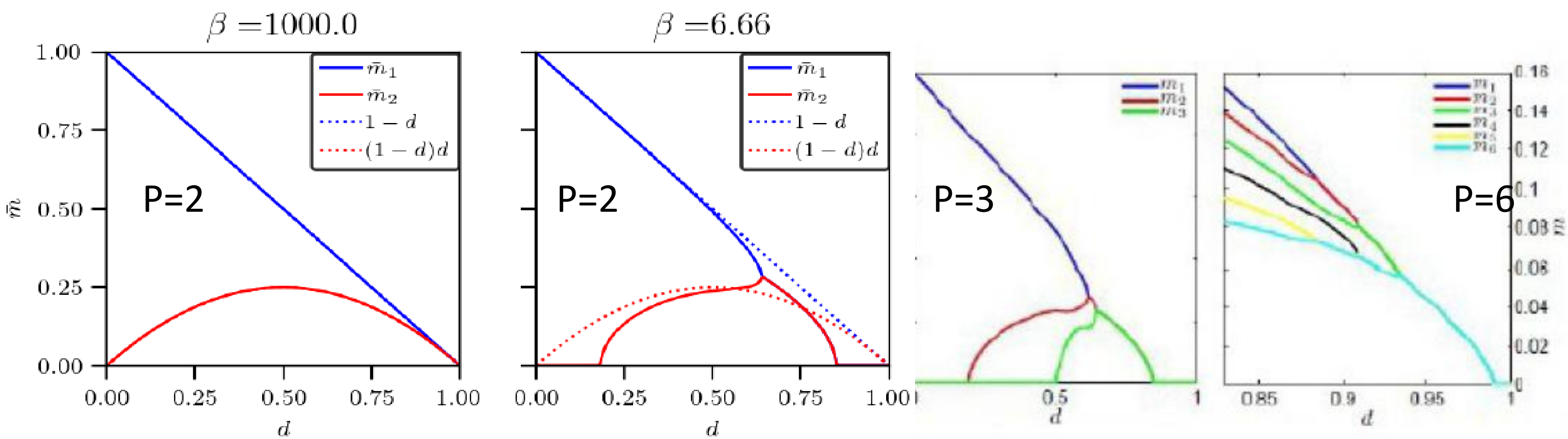
emergent parallel processing is the most convenient

$$E_{hp1} = -m_1^2 - m_2^2 = -(\frac{1}{2})^2 + random = -.25,$$

$$E_{hp2} = -m_1^2 - m_2^2 = -(\frac{1}{2})^2 - (\frac{1}{2})^2 = -.50.$$

$$m_1 = d(1-d)\tanh(\beta m_1) + \frac{(1-d)^2}{2}\{\tanh[\beta(m_1+m_2)] + \tanh[\beta(m_1-m_2)]\}, \qquad (2.6)$$

$$m_2 = d(1-d)\tanh(\beta m_1) + \frac{(1-d)^2}{2}\{\tanh[\beta(m_1+m_2)] - \tanh[\beta(m_1-m_2)]\} \qquad (2.7)$$



PRL 109, 268101 (2012) PHYSICAL REVIEW LETTERS week ending 28 DECEMBER 2012

Multitasking Associative Networks

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Extensive Parallel Processing on Scale-Free Networks

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Retrieval Capabilities of Hierarchical Networks: From Dyson to Hopfield

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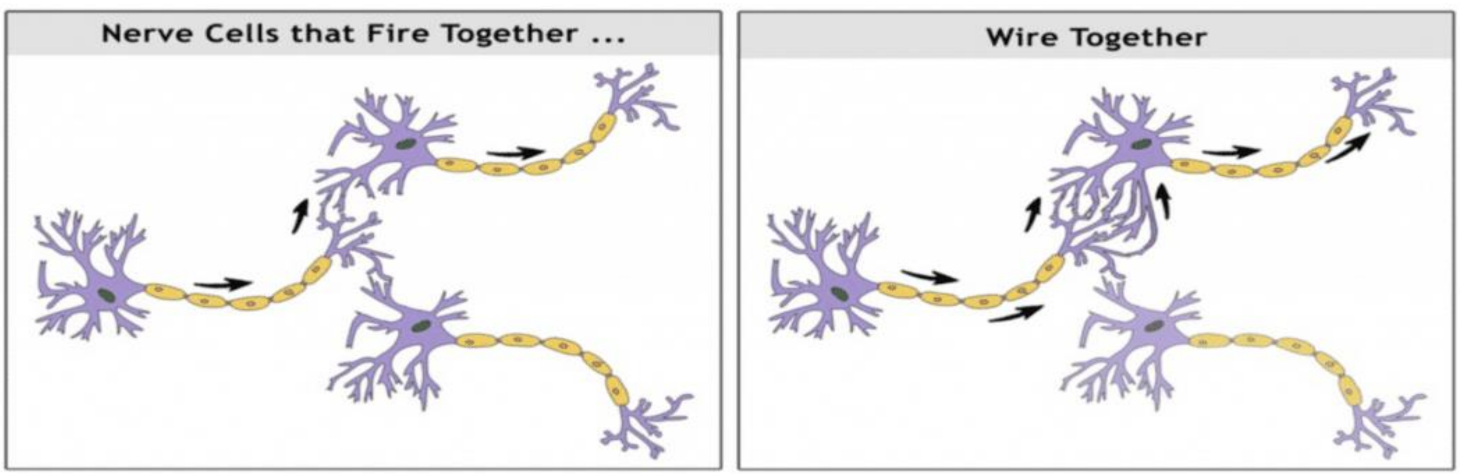
Pavlov's Classical Conditioning	Hebb's learning	
Correlations between concepts	Correlations between pairs of neurons	**Different Degrees of information**
Patterns of bits	Single bits	
Neutral stimuli associated with unconditioned stimuli	simultaneous activation of cells increases the synaptic strength.	**Learning by association**
Synapses store the correlations between concepts	Synaptic modification by correlated activity	**Synaptic modification**



Pavlov Classical Conditioning (not statistical mechanics)

Neural and Synaptic Dynamics

The evolution equation of the expectation value of the spins and synaptic matrix over time:

$$\bar{J}_{ij} := \sum_{\boldsymbol{\sigma}\in\Sigma_N}\sum_{\boldsymbol{J}\in\mathcal{J}_N}\mathscr{P}_N(\boldsymbol{\sigma},\boldsymbol{J},t)J_{ij}, \qquad \bar{\sigma}_i := \sum_{\boldsymbol{\sigma}\in\Sigma_N}\sum_{\boldsymbol{J}\in\mathcal{J}_N}\mathscr{P}_N(\boldsymbol{\sigma},\boldsymbol{J},t)\sigma_i$$

Dynamical equations

$$\tau\frac{d\bar{\sigma}_i}{dt} = \sum_{\boldsymbol{J},\boldsymbol{J}'\in\mathcal{J}_N}\sum_{\boldsymbol{\sigma},\boldsymbol{\sigma}'\in\Sigma_N}\sigma_i\left[\mathscr{P}_N(\boldsymbol{\sigma}',\boldsymbol{J}',t)w_{N,\beta}(\boldsymbol{J}',\boldsymbol{\sigma}'\to\boldsymbol{J},\boldsymbol{\sigma}) - \mathscr{P}_N(\boldsymbol{\sigma},\boldsymbol{J},t)w_{N,\beta}(\boldsymbol{J},\boldsymbol{\sigma}\to\boldsymbol{J}',\boldsymbol{\sigma}')\right]$$

$$\tau'\frac{d\bar{J}_{ij}}{dt} = \sum_{\boldsymbol{J},\boldsymbol{J}'\in\mathcal{J}_N}\sum_{\boldsymbol{\sigma},\boldsymbol{\sigma}'\in\Sigma_N}J_{ij}\left[\mathscr{P}_N(\boldsymbol{\sigma}',\boldsymbol{J}',t)w_{N,\beta}(\boldsymbol{J}',\boldsymbol{\sigma}'\to\boldsymbol{J},\boldsymbol{\sigma}) - \mathscr{P}_N(\boldsymbol{\sigma},\boldsymbol{J},t)w_{N,\beta}(\boldsymbol{J},\boldsymbol{\sigma}\to\boldsymbol{J}',\boldsymbol{\sigma}')\right]$$

Where τ and τ' are respectively the typical timescale for the neural and synaptic dynamics

After some tedious calculations we get *Note: self-consistency at equilibrium (lhs=0).*

$$\tau\frac{d\bar{\sigma}_i}{dt} = -\bar{\sigma}_i + \tanh\left(\beta\sum_j\bar{J}_{ij}\bar{\sigma}_j + \beta h_i\right), \qquad \tau'\frac{d\bar{J}_{ij}}{dt} = -\bar{J}_{ij} + \tanh\left(\beta\right)\bar{\sigma}_i\bar{\sigma}_j$$

Dynamical equations

$$\tau \frac{d\bar{\sigma}_i}{dt} = \sum_{\boldsymbol{J},\boldsymbol{J}' \in \mathcal{J}_N} \sum_{\boldsymbol{\sigma},\boldsymbol{\sigma}' \in \Sigma_N} \sigma_i \left[\mathcal{P}_N(\boldsymbol{\sigma}', \boldsymbol{J}', t) w_{N,\beta}(\boldsymbol{J}', \boldsymbol{\sigma}' \to \boldsymbol{J}, \boldsymbol{\sigma}) - \mathcal{P}_N(\boldsymbol{\sigma}, \boldsymbol{J}, t) w_{N,\beta}(\boldsymbol{J}, \boldsymbol{\sigma} \to \boldsymbol{J}', \boldsymbol{\sigma}') \right]$$

$$\tau' \frac{d\bar{J}_{ij}}{dt} = \sum_{\boldsymbol{J},\boldsymbol{J}' \in \mathcal{J}_N} \sum_{\boldsymbol{\sigma},\boldsymbol{\sigma}' \in \Sigma_N} J_{ij} \left[\mathcal{P}_N(\boldsymbol{\sigma}', \boldsymbol{J}', t) w_{N,\beta}(\boldsymbol{J}', \boldsymbol{\sigma}' \to \boldsymbol{J}, \boldsymbol{\sigma}) - \mathcal{P}_N(\boldsymbol{\sigma}, \boldsymbol{J}, t) w_{N,\beta}(\boldsymbol{J}, \boldsymbol{\sigma} \to \boldsymbol{J}', \boldsymbol{\sigma}') \right]$$

Where τ and τ' are respectively the typical timescale for the neural and synaptic dynamics

After some tedious calculations we get

Note: self-consistency at equilibrium (lhs=0).

$$\tau \frac{d\bar{\sigma}_i}{dt} = -\bar{\sigma}_i + \tanh\left(\beta \sum_j \bar{J}_{ij}\bar{\sigma}_j + \beta h_i\right), \qquad \tau' \frac{d\bar{J}_{ij}}{dt} = -\bar{J}_{ij} + \tanh\left(\beta\right)\bar{\sigma}_i\bar{\sigma}_j$$

Discrete dynamical equations

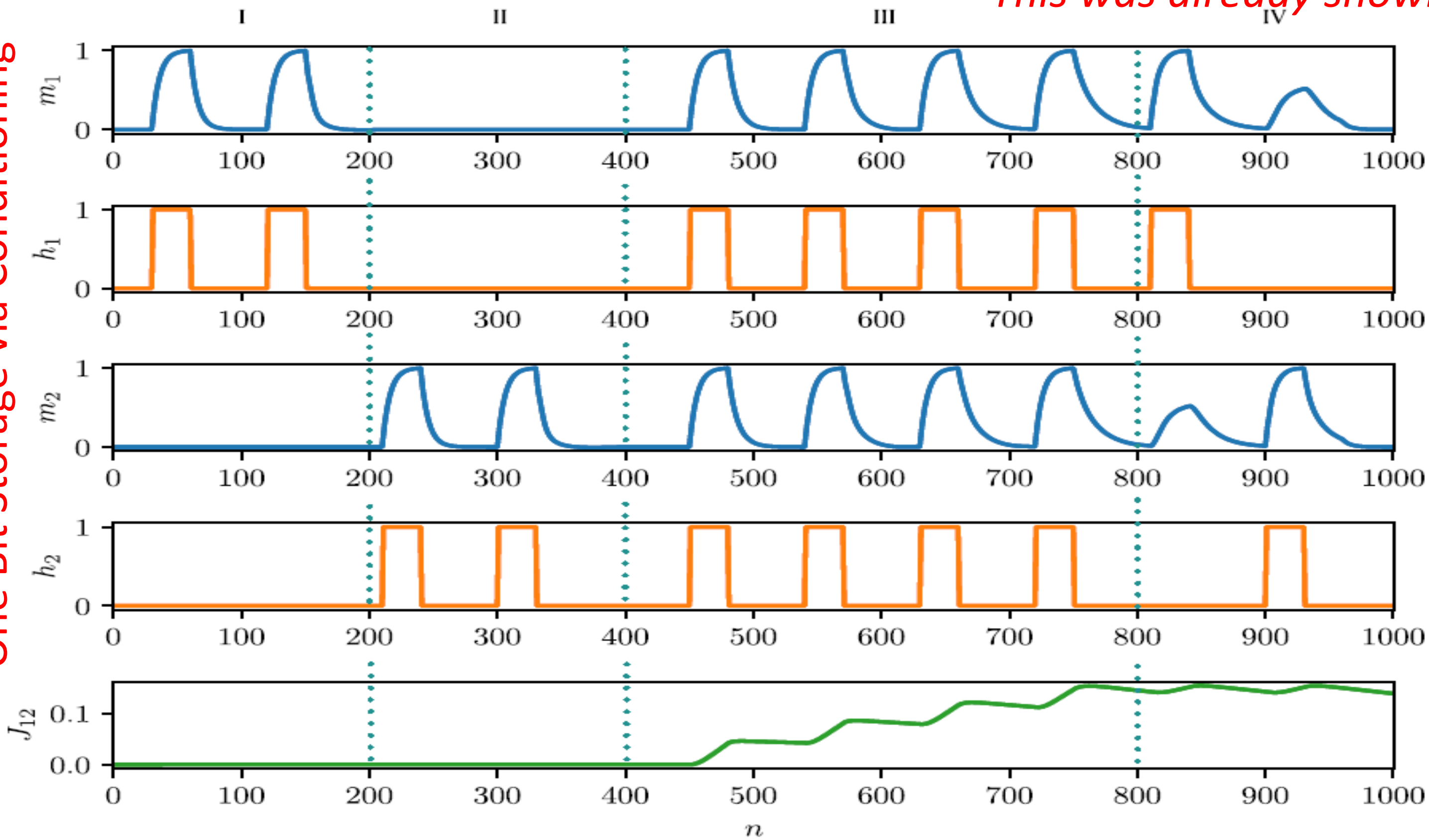
$$\bar{\sigma}_i^{(n+1)} = \bar{\sigma}_i^{(n)}\left(1 - \frac{\delta t}{\tau}\right) + \frac{\delta t}{\tau}\tanh\left(\beta \sum_{i \neq j}^{N} \bar{J}_{ij}^{(n)}\bar{\sigma}_j^{(n)} + u\beta h_i^{(n)}\right) \qquad \bar{J}_{ij}^{(n+1)} = \bar{J}_{ij}^{(n)}\left(1 - \frac{\delta t}{\tau'}\right) + \frac{\delta t}{\tau'}\bar{\sigma}_j^{(n)}\bar{\sigma}_i^{(n)}\tanh\beta,$$

We can choose $\delta t = c\tau, \quad c \in (0,1]$ such that the dynamics depends on c and τ/τ'

$$\tau \frac{d\bar{\sigma}_i}{dt} = -\bar{\sigma}_i + \tanh\left(\beta \sum_j \langle J_{ij} \rangle \bar{\sigma}_j + \beta h_i\right) \qquad \tau' \frac{d\bar{J}_{ij}}{dt} = -\bar{J}_{ij} + \tanh(\beta)\, \bar{\sigma}_i \bar{\sigma}_j .$$

This was already shown by Guerra in 1990.

One Bit Storage via Conditioning



$\tau \sim 1\, ms$

$\frac{\tau}{\tau'} \ll 1$

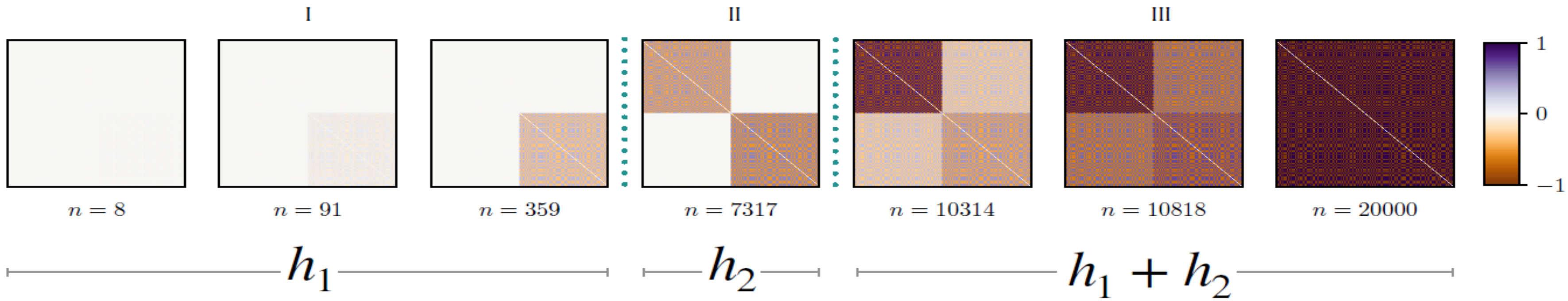
$\tau' \sim h$

At the single-bit level Hebb's prescription is recovered. Now we move to concepts (whose length is N/2) to inspect Pavlov!

$$\boldsymbol{\xi}^1 = \left(\xi_1^1, \xi_2^1, \ldots, \xi_{\frac{N}{2}}^1, 0, 0, \ldots, 0\right),$$

$$\boldsymbol{\xi}^2 = \left(0, 0, \ldots, 0, \xi_{\frac{N}{2}+1}^2, \xi_{\frac{N}{2}+2}^2, \ldots, \xi_N^2\right).$$

Emergence of concept's correlations by Classical Conditioning



II:

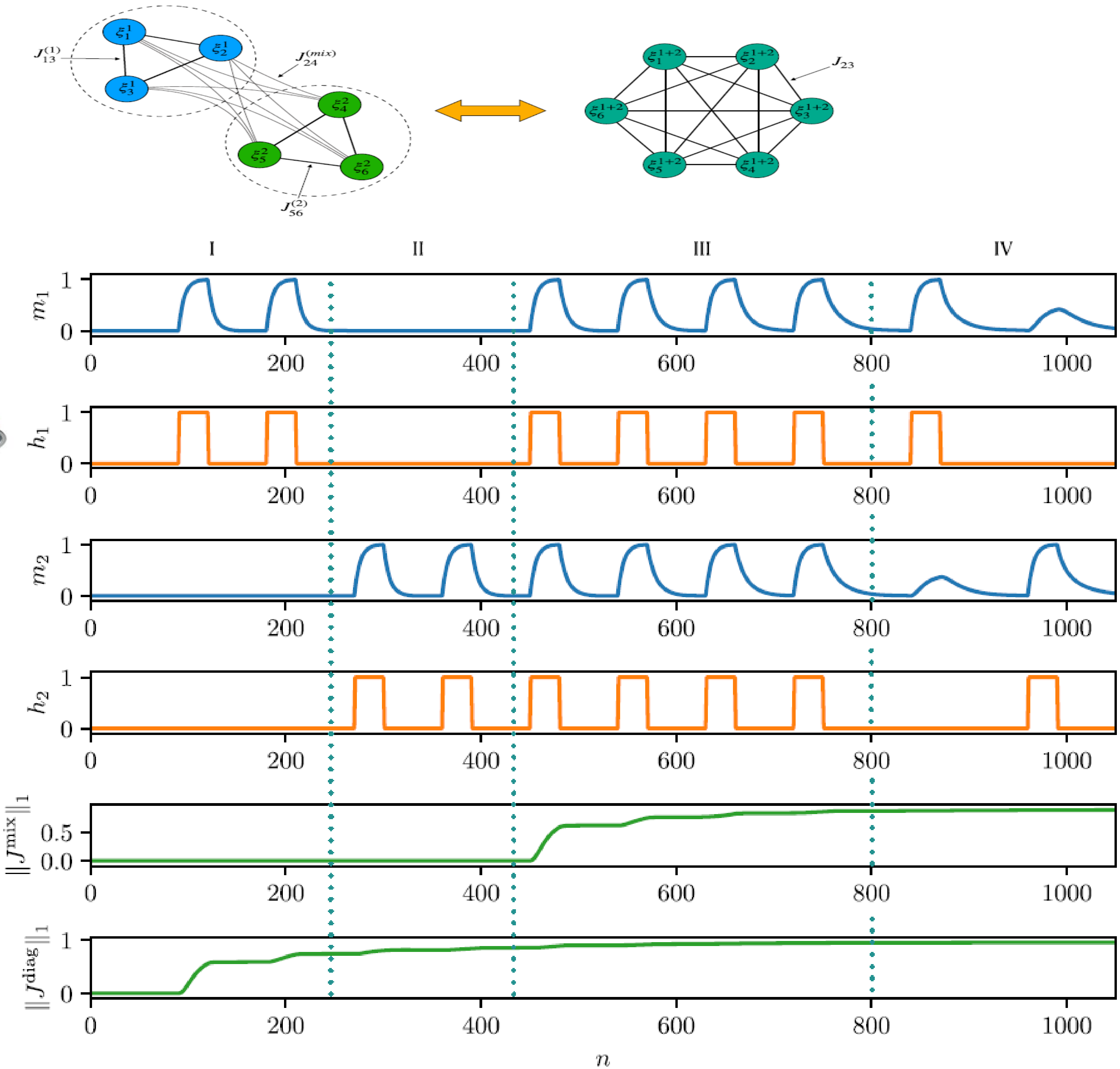
$$\boldsymbol{J}(\{\boldsymbol{\xi}^1,\boldsymbol{\xi}^2\}) = \frac{1}{2}\times\begin{bmatrix}\boldsymbol{J}^1 & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{J}^2\end{bmatrix}, \quad J_{ij}(\{\boldsymbol{\xi}^1,\boldsymbol{\xi}^2\}) = \frac{1}{2}\times\begin{cases}\xi_i^1\xi_j^1 & (i,j)\in B_1\\ \xi_i^2\xi_j^2 & (i,j)\in B_2\\ 0 & (i,j)\vee(j,i)\in B_{mix}\end{cases}.$$

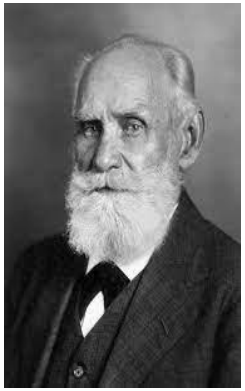
$$B_1 = [1,N/2]\times[1,N/2],\, B_2 = [N/2+1,N]\times[N/2+1,N],\, B_{mix} = [1,N/2]\times[N/2+1,N]\,.$$

III:

$$\boldsymbol{J}(\{\boldsymbol{\xi}^1,\boldsymbol{\xi}^2\}) = \frac{1}{2}\times\begin{bmatrix}\boldsymbol{J}^1 & \boldsymbol{J}^{(\mathrm{mix})}\\ \boldsymbol{J}^{(\mathrm{mix})} & \boldsymbol{J}^2\end{bmatrix}, J_{ij}(\boldsymbol{\xi}^{1+2}) = \begin{cases}\xi_i^1\xi_j^1 & (i,j)\in B_1\\ \xi_i^2\xi_j^2 & (i,j)\in B_2\\ \xi_i^1\xi_j^2 & (i,j)\in B_{mix}\\ \xi_j^1\xi_i^2 & (j,i)\in B_{mix}\end{cases}.$$

The synaptic matrix converges to the temporal mean of the stimuli $\bar{J}_{ij}(t) \approx \frac{1}{t}\int_0^t \xi_i(t')\xi_j(t')dt'$


I
II
III
IV
m_1
h_1
m_2
h_2
$\|J^{\mathrm{mix}}\|_1$
$\|J^{\mathrm{diag}}\|_1$
n

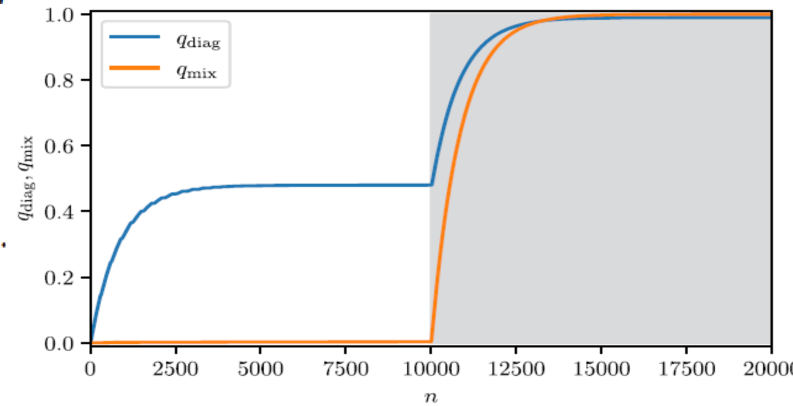


Point Three: Classical Conditioning among two concepts. Guerra's argument on multi-tasking nets.

$$q_{\text{diag}}^{(n)} := \frac{1}{2}\sum_{k=1}^{2} \frac{\sum_{(i,j)\in B_k} J_{ij}^{k} J_{ij}^{(n)}}{\sum_{(l,m)\in B_k} J_{lm}^{k} J_{lm}^{k}},$$

$$q_{\text{mix}}^{(n)} := \frac{\sum_{(i,j)\in B_{\text{mix}}} J_{ij}^{\text{mix}} J_{ij}^{(n)}}{\sum_{(l,m)\in B_{\text{mix}}} J_{lm}^{\text{mix}} J_{lm}^{\text{mix}}}.$$

As for the off-diagonal blocks, since there are no correlations among the related neurons, they contain no information. Next, we present both stimuli simultaneously: the overall stimulation provided to the network is



$$\boldsymbol{\xi}^{1+2} = \boldsymbol{\xi}^1 + \boldsymbol{\xi}^2 = (\xi_1^1, \xi_2^1, \ldots, \xi_{\frac{N}{2}}^1, \xi_{\frac{N}{2}+1}^2, \xi_{\frac{N}{2}+2}^2, \ldots, \xi_N^2),$$

and it is retained long enough to allow synapses to be plastic. synaptic matrix converges to

$$J(\boldsymbol{\xi}^{1+2}) = \begin{pmatrix} J^1 & J^{\text{mix}} \\ J^{\text{mix}} & J^2 \end{pmatrix}, \qquad (3.8)$$

$$\xi^1 = (a, b, 0, 0)$$

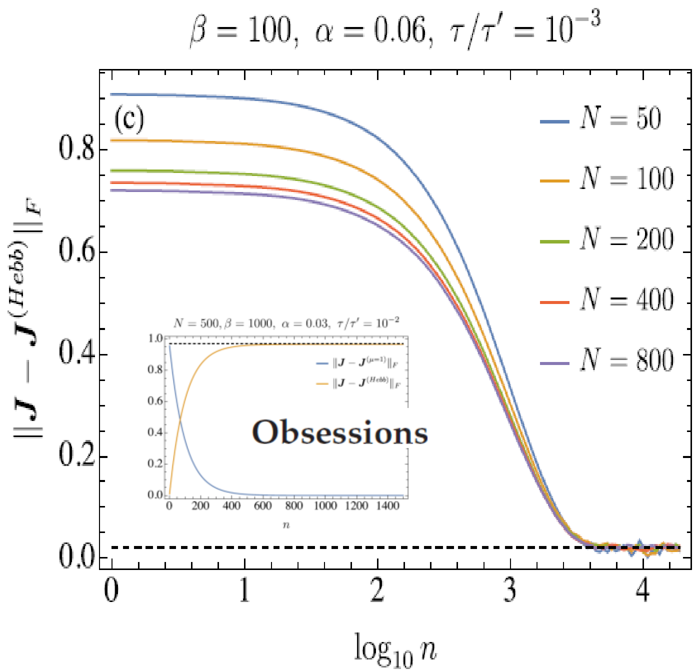
$$\xi^2 = (0, 0, c, d)$$

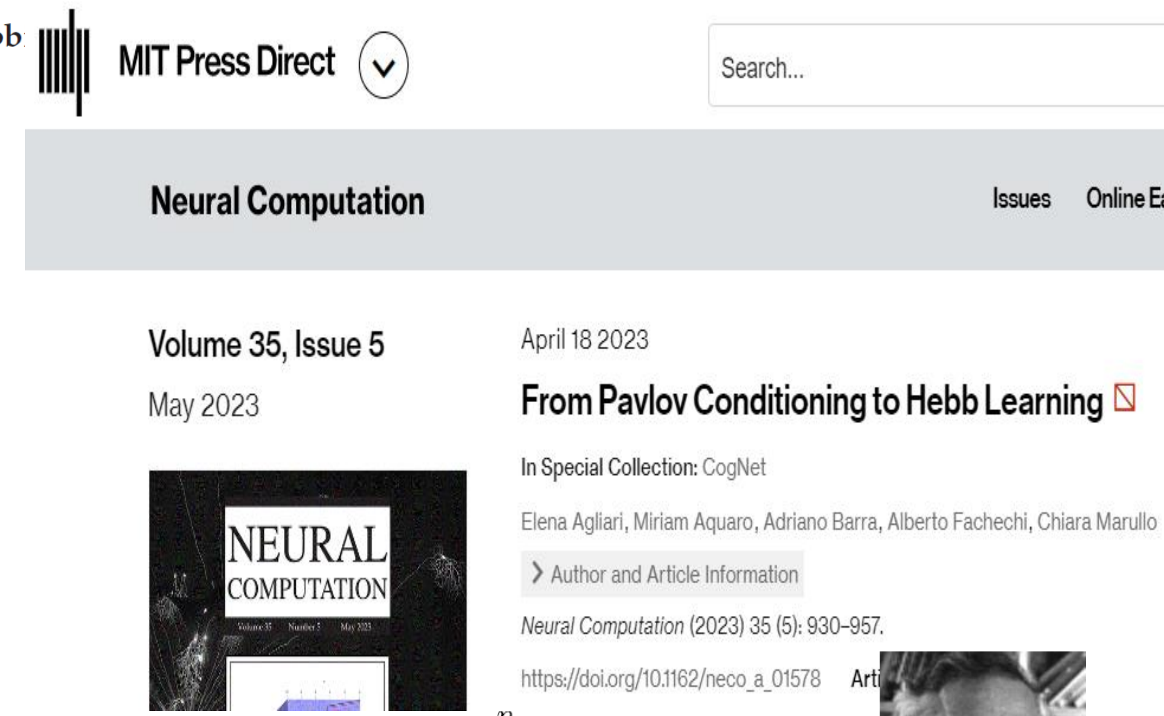
$$J_{ij}(\boldsymbol{\xi}^{1+2}) = \begin{cases} \xi_i^1 \xi_j^1 & (i,\ j) \in B_1 \\ \xi_i^2 \xi_j^2 & (i,\ j) \in B_2 \\ \xi_i^1 \xi_j^2 & (i,\ j) \in B_{mix} \\ \xi_j^1 \xi_i^2 & (j,\ i) \in B_{mix}. \end{cases}$$

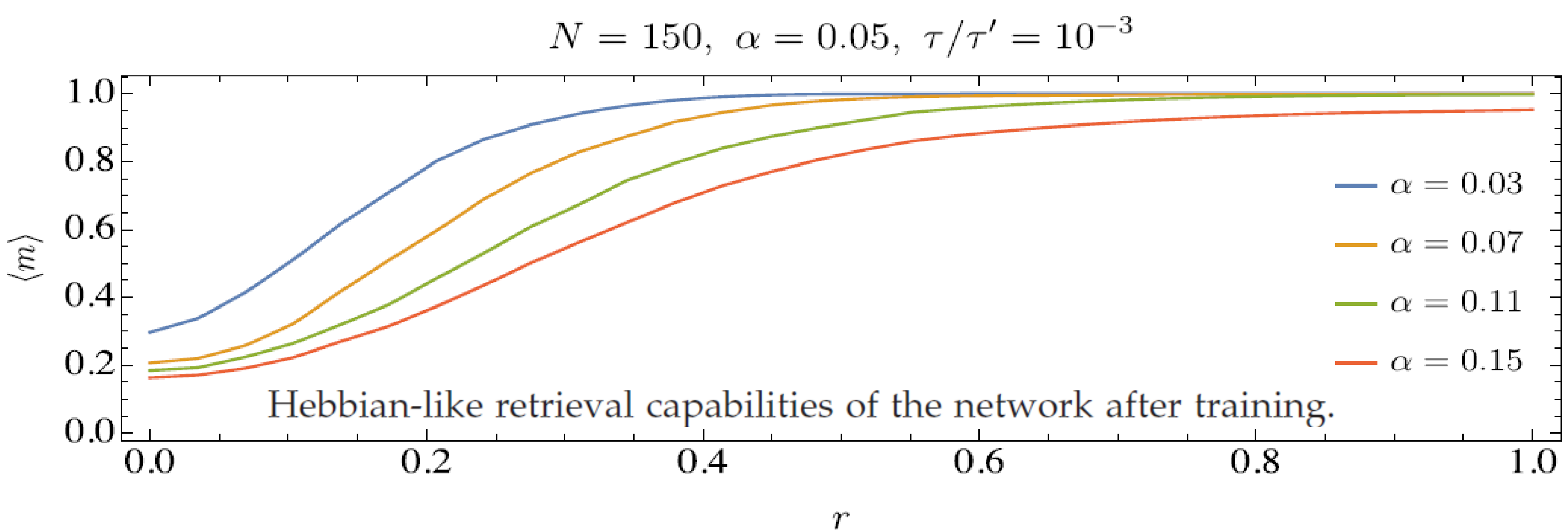
$$\xi^C = (a, b, c, d)$$

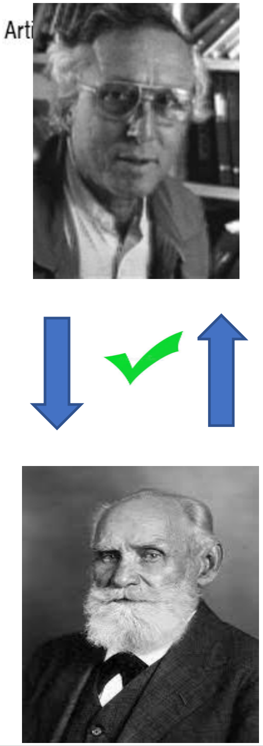
$$J_{Hebb} = \xi_i^C \xi_j^C \rightarrow \begin{bmatrix} 0 & ab & ac & ad \\ ba & 0 & bc & bd \\ ca & cb & 0 & cd \\ da & db & dc & 0 \end{bmatrix}$$

Convergence of the Synaptic Dynamics to the Hebb









acknowledgement

Brain-inspired ULtra-fast & ULtra-sharp machines for AI-assisted health-care
MAECI-MOST funded projects 2021 (time window 2022-2023)

Israeli Partner: Prof. Ido Kanter (Head), Department of Physics, Bar-Ilan University

Italian Partner: Prof. Adriano Barra (Head), Dipartimento di Matematica & Fisica, Università del Salento

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THANK YOU FOR YOUR GENTLE ATTENTION!

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