Discrete and Differential Geometry A conference to celebrate the seventieth birthday of Prof.s M. Bilotti and D. Perrone Book of Abstracts

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Contents

S. DRAGOMIR (Università della Basilicata)

Fifty years of contact Riemannian manifolds and f-structures with complemented frames versus Cauchy-Riemann geometry

Abstract: We review the basics of contact Riemannian geometry, to which a significative portion of the scientific creation by Domenico Perrone is devoted. We demonstrate the construction of natural almost CR structures $T_{1,0}(M)$ on contact Riemannian manifolds $\big(M, (\phi, \xi, \eta, g)\big)$ and characterize their integrability in terms of the Tanno tensor and as related to the tangential Cauchy-Riemann pseudocomplex. For any contact Riemannian manifold M, the lack of integrability of $T_{1,0}(M)$ is circumnavigated by exploiting the wealth of additional geometric structure, an approach leading to contact metric analogs to objects familiar within Webster's pseudohermitian geometry e.g. Tanaka-Webster connection, sublaplacians, and Fefferman's metric F_η, a Lorentzian metric on the total space $C(M)$ of the canonical circle bundle over M. We show that $C(M)$ carries a natural f-structure with two complemented frames got as the covariant derivative of a null Killing vector field on $(C(M), F_\eta)$.

A. FINO (Università di Torino)

An overview on G_2-structures in relation with contact geometry and geometric flows

Abstract: A G_2-structure on a seven-dimensional manifold M is characterized by positive 3-form φ. The 3-form φ induces and orientation and a Riemannian metric g_φ. The intrinsic torsion of a G_2-structure φ can be identified with the covariant derivative $\nabla\varphi$, where ∇ is the Levi Civita connection of the Riemannian metric g_φ. It vanishes identically if and only if both $d\varphi = 0$ and $d *_\varphi \varphi = 0$, where $*_\varphi$ denotes the Hodge operator of g_φ. When this happens, the G_2-structure is said to be torsion-free, its associated Riemannian metric g_φ is Ricci-flat and the corresponding Riemannian holonomy group is a subgroup of the exceptional Lie group G_2. G_2-structures can be divided into classes, which are characterized by the expression of the exterior derivatives $d\varphi$ and $d*_\varphi\varphi$ 24]. In particular, a G_2-structure φ is called closed (or calibrated) if $d\varphi = 0$, while it is called coclosed (or cocalibrated) if $d*_\varphi\varphi = 0$. Since the Ricci tensor and the scalar curvature of the metric induced by a G_2-structure can be expressed in terms of the intrinsic torsion, it may happen that certain restrictions on the curvature give rise to some constraints on the intrinsic torsion. Although every 7-manifold with G_2-structure admits an almost contact structure, in dimension 7 contact geometry and G_2 geometry have distinguished properties which are rather different than those in the other. In contact geometry there are no local invariants, in other words, every contact 7-manifold is locally contactomorphic to $\mathbb{R}^7$ equipped with the standard contact structure. On the other hand, in G_2 geometry it is the G_2 structure itself that determines how local neighborhoods of points look like, and as a result, manifolds with G_2 structures can look the same only at a point. In the first part of the talk I will describe some relations between G_2-structures and almost contact geometry, reviewing some results obtained by Domenico Perrone in contact geometry. In the second part I will present some recent results on geometric flows of closed and coclosed G_2-structures.

J.W.P. HIRSCHFELD (University of Sussex)

Mauro Biliotti

Abstract: Mauro Biliotti has had a distinguished career in Mathematics. From 1975, he has published 77 papers, many on his own and also with 11 co-authors. The topics covered have ranged all parts of the theory of affine and projective planes. Some of his work is reviewed.

G. KORCHMAROS (Università della Basilicata)

Geometric structures with many symmetries

Abstract: Permutation groups have played a fundamental role in many investigations concerning Finite geometries, Graph theory, and, in recent years, algebraic curves over finite fields. The huge amount of work done in this area has greatly been stimulated by the following objectives:
(i) Construction of nice finite geometric structure from permutation group.
(ii) Characterizations of geometric structures by their automorphism groups.
(iii) Classifications of large automorphism groups of geometric structures.
In our talk we focus on geometric structures on which a large automorphism group can act. A key issue is to understand the constraints imposed by the underlying geometry on the structure and action of its automorphism groups. As a matter if fact, even relevant geometric structures may happen to impose drastic restrictions so that their automorphism groups are either trivial or of small order. We show that ovals and unitals in a projective plane are geometric structures where the interplay between geometry and permutation groups works well and gives substantial contributes to the above questions. We also give an overview of the recent developments in the study of algebraic curves over a finite field which have many automorphisms with respect to their genera and Hasse-Witt invariants. Geometry and combinatorics together with deeper results from Group theory (especially on permutation groups) have been an essential tool in improving previous results obtained by classical methods based on Algebraic geometry and Function field theory.

G. LUNARDON (Università di Napoli “Federico II”)

Strutture di traslazione

Abstract: Si presentano alcune applicazioni della generalizzazione del teorema di isomorfismo tra spazi di Sperner, dimostrato da Biliotti, Bonetti e Lunardon. Dopo aver illustrato la ovvia generalizzazione alle strutture di traslazione con gruppo delle traslazioni abeliano, si ottiene un criterio di isomorfismo per i quadrangoli generalizzati di traslazione associati ad un flock del cono quadratico. Infine, si dimostra che ogni codice di tipo MRD de nisce una struttura di traslazione e, utilizzando il citato teorema d’isomorfismo, si costruiscono alcuni invarianti per tali codici.

E. MUSSO (Politecnico di Torino)

Contact and Cauchy-Riemann geometry of a transversal knot in the 3-sphere

Abstract: The first part of the talk is about 3-dimensional contact geometry, the Bennequin number of transverse knots and the Fuchs-Tabachnikov conjecture. In the second part we recall the notion of Cauchy-Riemann structures on 3-dimensional contact manifolds. We illustrate the canonical Cauchy-Riemann structure on the 3-sphere and the associated Fefferman fibration. In the last part, we consider the CR-geometry of generic transverse knots and we define three global invariants: the pseudo-conformal spin, the Maslov index and the Cauchy-Riemann self-linking number. We analyze the interrelations between these invariants and the Bennequin number. Subsequently, we explain two results about critical knots of the CR-strain functional, that is, the lower-order cr-invariant variational problem for generic transverse knots. We end with some open problems.