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An evolutionary game model for behavioral gambit of loyalists: Global awareness and risk-aversion

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Abstract – We study the phase diagram of a minority game where three classes of agents are present. Two types of agents play a risk-loving game that we model by the standard Snowdrift Game. The behaviour of the third type of agents is coded by *indifference* with respect to the game at all: their dynamics is designed to account for risk-aversion as an innovative behavioral gambit. From this point of view, the choice of this solitary strategy is enhanced when innovation starts, while is depressed when it becomes the majority option. This implies that the payoff matrix of the game becomes dependent on the global awareness of the agents measured by the relevance of the population of the indifferent players. The resulting dynamics is nontrivial with different kinds of phase transition depending on a few model parameters. The phase diagram is studied on regular as well as complex networks.

Introduction. – Risk and uncertainty are important elements of human behavior, in particular when dealing with interactions of international teams and global counterparts. Here, by risk we just mean the intrinsic uncertainty in a multi-strategy individual interaction/game characterized by a pay-off depending nontrivially on the mutual strategic choices. The level of risk or, generally speaking, uncertainty that may be tolerable is not constant over different cultures, see for instance [1,2]. It appears that the risk aversion attitude is correlated with a low amount of tolerance for vague or ambiguous behaviours. In such a case, structured sets of rules are preferred over more undetermined situations. This implies a sort of cultural payoff component associated with playing risk-averse strategies. Besides, cultural conformity is a key factor in the evolution of complex culture in humans [3–5] as well as in animals [6]. On the other hand, there is tension with the drive to innovate. Innovation is an important ingredient to promote longstanding high levels of welfare, as it happens in real markets [7–9]. Deviation from conformity is an important evolutive mechanism, the so-called behavioral gambit, see for instance [10]. In this paper we propose a simple model describing how innovation is triggered when a population component is risk-averse in the presence of many individuals accepting playing some risky competition requiring cooperation. We shall assume a tendency to play risk-averse strategy with enhanced payoff for the above-mentioned cultural reasons. On the other hand, we shall also assume that an increased sense of safety due to a large number of risk-averse players will feedback the strategy choice and reduce the extra payoff allowing for more risk-loving attitudes. In the context of epidemics disease critical spreading [11], it is also natural to assume that the effective infection probability is affected by the perception of the risk of being infected. This has been assumed to be related to the fraction of infected neighbors in the recent models discussed in [12–14]. The precise mechanisms for this to happen are an important issue, like aggregation effects [15] or the role of the endorsers [16]. Here, we shall explore the dynamical balance between risk-aversion and the opposite behaviour induced by global awareness of the fact that such attitude is the majority option: a natural application of the present model deals with the phenomenon of *nonconformism*, a tendency ranging from social and political settings to economic or financial contexts. Nonconformist individuals refuse to take part in the game, and their behavior does not depend on the interactions with other competitors. They also tend to arrange in closed subcommunities sharing the same behavior (or opinion). A distinctive feature of the phenomenon is that the nonconformist option is discouraged if this behavior becomes a mainstream mindset.

We shall describe this competition in the context of evolutionary game theory [17–20]. This is a theoretical framework where emergence of collective behaviour may be observed, especially when played on structured populations [21,22]. We shall adopt the Snowdrift Game (SG) as our background model of competition [23]. In this model [24], cooperating players can coexist with defector ones even in well-mixed systems. The spatial structure is important and may even hinder the cooperative attitude [25]. In addition, Santos and Pacheco [26] discovered that cooperation generically emerges on scale-free networks, while Szabó *et al.* [27–29] presented a model accounting for a limited rationality of players in the game with the attempt to frame real system dynamics.

We consider the memory-based Snowdrift Game proposed in [30], in which the finite individual rationality affects the system dynamics since the adoption of a given strategy depends on the information already stored in the players' memory. This approach is based on [31], where a so-called *minority game* [32] is considered, where the individual experience crucially affects the system dynamics [33,34].

To take into account the risk-averse strategy option we included *solitary players* [35,36]. In the standard snowdrift game with these loners, one has a solitary-strategy describing people choosing not to participate in the competition and would prefer to take a fixed albeit small payoff. Hauert *et al.* studied the effects of their presence in a generalization of the prisoner dilemma game called the *public goods game* [28,37]. This basic model will be modified in order to take into account two novel mechanisms according to the previous discussion. The first is a cultural payoff enhancements for those players choosing to avoid the SG strategies. The second is a damping effect of the risk-aversion tendency. It depends on the global awareness of the system, *i.e.* on the fraction of players preferring the risk-averse strategy, in the spirit of [12].

The modified snowdrift game. – In [35] the authors resumed the SG model with the loners in the evolutionary setting, while memory in the system has been considered for the pure SG in [38]. The main results of these two papers will be summarized in the Supplementary Material `Supplementarymaterial.pdf` (SM).

Our modification of the Snowdrift Game with S-players is based on the payoff matrix

$$\mathbf{P} = \begin{array}{c} \\ \mathbf{C} \\ \mathbf{D} \\ \mathbf{S} \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{C} & \mathbf{D} & \mathbf{S} \end{array} \\ \begin{pmatrix} 1 & 1-r & q \\ 1+r & 0 & q \\ (1+\xi)\,q & (1+\xi)\,q & (1+\xi)\,q \end{pmatrix} \end{array}, \tag{1}$$

where r is the cost-to-reward ratio, q a fixed constant and at each round

$$\xi = 1 - f_S, \tag{2}$$

f_S being the fraction of players in the S-strategy. The payoff for pair of strategies involving solitaries is standard for $\xi = 0$. Adopting the solitary strategy has an additional payoff $\xi\,q$ decreasing when the number of S-players increases. The rules of this memory-based evolutionary game are the following: let us consider a generic graph; each player lies on a node of this graph such that pairwise connected players challenge reciprocally match by match and this happens for all the (connected) couples. The total payoff of any player is simply the linear sum of the payoff collected in each single duel. Once a round is terminated, all the players evaluate their performances by playing a virtual match where they use the anti-strategy with respect to the one they've just adopted in order to calculate their potential reward with this revised settings: if the latter is actually better than the employed one (*i.e.*, if the player obtained higher score with the anti-strategy), it keeps the latter as the best strategy to be used and it stores this information in its memory (whose length is fixed to M bits[1]). In the successive step all players update their memory at once by adding a new strategy extracted with probability given by the frequency of the already stored C, D and S choices and the dynamics then takes place by iterating this procedure.

Note that the parameter ξ introduces a global feedback in the score of the S-players, which now depends not only on the local interaction but also on the global state of the community.

We introduce, as order parameters, the three fractions of players

$$f_C \equiv \frac{C}{N}, \qquad f_D \equiv \frac{D}{N}, \qquad f_S \equiv \frac{S}{N},$$

(where $N = C + D + S$), and we will study them as functions of r for fixed values of $q = \{0.1, \dots, 0.4\}$ and viceversa[2].

Note further that a pure SG with memory-driven evolution has been considered in [38], both in regular lattices and complex networks, where it has been shown that taking the memory Markovian and with finite length, the systems evolves towards a (almost) stable state with a roughly constant density of cooperators f_C. In the particular case of regular lattices, cooperators and defectors tend to group forming well-defined spatial clusters. Moreover, the density of cooperators shows a *step-shape* for varying r, with the number of steps determined by the cardinality of the neighborhoods. A variation of the SG theme with the S-players has been instead analyzed in [35], where the authors focused on completely connected graphs and regular (with four neighbors) lattices. Their main results are that,

[1] We analyze in depth this point in the SM.

[2] Numerically, the curves are obtained averaging over 100 Monte Carlo simulations for any value of r and q.

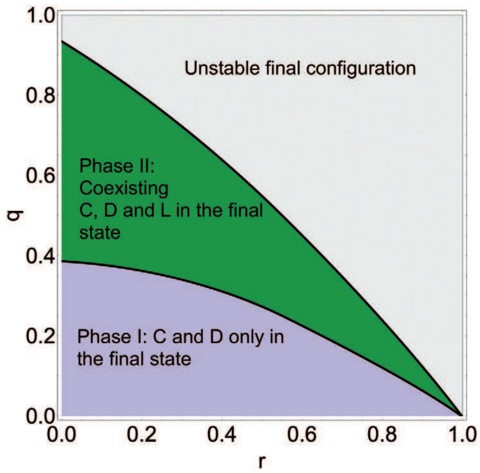

Fig. 1: (Color online) Phase diagram of the model. The black lines mark the transitions from the first phase without solitaries to the second with the three strategies coexisting and from the second to the (unstable) third one.

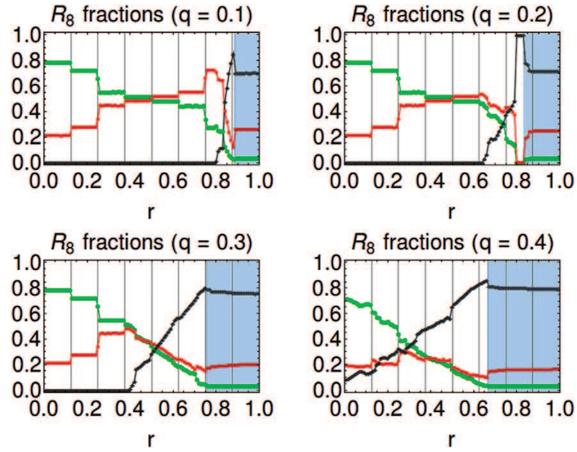

Fig. 2: (Color online) Fractions of cooperators f_C, defectors f_D and solitary players f_S as functions of the parameter r for $q = 0.1, \ldots 0.4$. Green dots denotes f_C, red f_D and gray f_S. The shaded regions in the plots correspond to the third phase of the system where no stable spatial arrangement is reached. Gray vertical lines correspond to the phase transitions in the SG without S-players. Further analysis is presented in the SM.

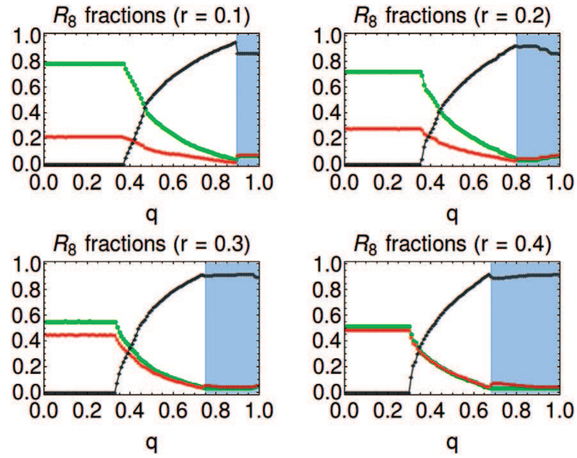

Fig. 3: (Color online) Fractions of cooperators f_C, defectors f_D and solitary players f_S as functions of the parameter q for $r = 0.1, \ldots 0.4$. Green dots denotes f_C, red f_D and gray dots f_S. The shaded regions in the plots correspond to the third phase of the system where no stable spatial arrangement is reached.

in a completely connected graph, the coexistence of the three characters is impossible: C- and D-players can exist only in the absence of solitary category, or the latter take over the entire population. For the regular graph, instead, the presence of the S-players leads to an improvement of the collaborative attitude, coexistence is possible, and in general the C-players can survive in the full parameter space of the model.

Regular lattice R_8**.** – In this section we discuss the results for our extension in a regular lattice with 8 neighbors. The phase diagram of the model can be divided into three main regions in the $(r, q) = [0, 1] \times [0, 1]$ square, as shown in the phase diagram presented in fig. 1. In the first and second phases the system evolves to a steady state with coexisting C-D and C-D-S players respectively. In the third phase, the players never reach a stable strategy choice. In the first phase (highlighted in blue), for r and q small enough, the system evolves toward a stable final state without S-players. The final state is the same as it would be for a community without the S-players at all, and the step-like shapes of the $f_{C,D}$ fractions are evident from the plots of the densities *vs.* r as reported in fig. 2. In the second phase (highlighted in green), the system evolves almost everywhere (see below) to a stable final configuration characterized by the coexistence of the three kinds of players. However the total fraction of S individuals, as the size of their clusters, increase both with q and r, as a glance at both figs. 2, 3 may confirm.

This phase is characterized by typical spatial configurations, where S-players arrange forming clustered sub-communities. We stress that this stable spatial arrangement of the strategies is strikingly different from the results of [35]. Both the stability of the final state and the regularity of the patterns are consequences of the memory-based update for the strategies of the players. Despite the phase diagram of the model of [35] is qualitatively similar to the present one, the probabilistic update of their players forbids the stability of the choices and consequently the formation of the patterns. The rearrangement in sub-communities by the S-players is instead dictated by the nonlinear feedback of ξ in the payoff matrix, favouring the grouping of players that prefer not to participate.

Looking at the various $q = 0.1, 0.2, \ldots$ in fig. 2, one can clearly see that once the solitaries appear, their fraction becomes quickly dominant increasing r: a higher payoff for the defectors means a higher risk to be cheated for the C-players, and progressively more and more individuals find advantageous to avoid this risk, choosing for the solitary behavior.

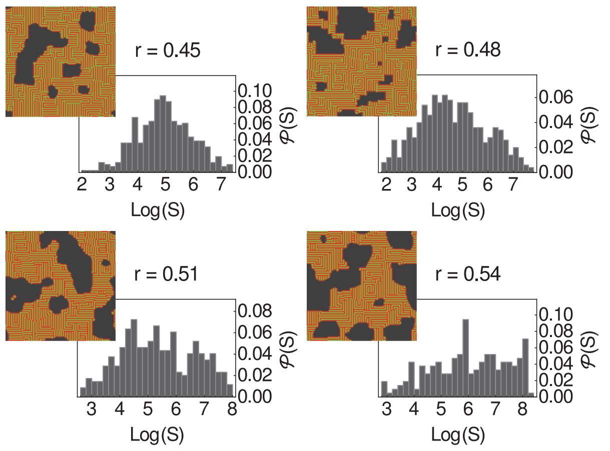

Fig. 4: (Color online) Probability distributions (and relative snapshots) for the size S of S-clusters (in gray), in a 100×100 lattice for fixed $q = 0.3$ and varying r around the percolation threshold for a giant S-component.

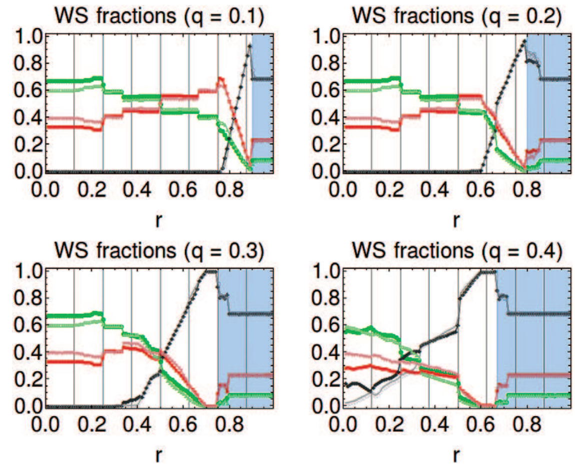

Fig. 5: (Color online) Fractions of cooperators f_C, defectors f_D and solitary players f_S as functions of the parameter r, for a WS network with $\theta = 0.1$ (darker colors) and $\theta = 0.5$ (lighter colors). Green dots denote f_C, red f_D and gray f_S. The shaded regions in the plots correspond to the third phase of the system, where no stable spatial arrangement is reached.

The fraction f_S is an increasing function of both r and q but in rather different ways: while the transitions of f_S in r are first-order (*i.e.*, there is a genuine *jump* by the order parameter at the critical values r_c, see fig. 2)[3], the transition from zero to nonzero values for f_S *vs.* q resembles a critical (second-order) scenario: the order parameter continuously raises from zero at q_C, see fig. 3 (further there is a genuine divergence in the susceptibility —the variance of the order parameters, suitably amplified by a volume factor— for $q \to q_C$, as closely examined in the SM).

We aim to better characterize phase II (since understanding network's dynamics in that phase is crucial to see why there is a third, unstable, phase in the phase diagram) by looking at the evolution of the size distribution of the S-clusters: in fig. 4 we show the probability distributions for the size of S-clusters, in a 100×100 lattice at fixed $q = 0.3$ and varying r around the percolation threshold for a S-players' giant component. These loners appears in a relatively large number of small islands progressively increasing their size and eroding the regular pattern of C- and D-players, eventually merging. As anticipated above, in this phase the system does not always reach a perfectly stable final configuration: this is because a (relatively small) fraction of players (typically located at the borders of the S-clusters) can exhibit an oscillatory behavior. These local instabilities are closely related to an oscillating balance in the payoff matrix among the possible gains, due to the nonlinear feedback of ξ: this results in a perpetual indecisiveness for some players as a structural property for models with global awareness.

The growth of the S-density leads eventually to another transition[4] toward the third phase, where the system is intrinsically unstable: all the players are never able to make a definitive choice for a strategy. Indeed, for large enough q, the incentive not to participate to the SG would make the S-strategy the preferred choice for all the individuals and the system would evolve toward a state where the S-players completely dominate the system. Nevertheless, a configuration with only S-players is not stable. With $f_S = 1$ and $\xi = 0$ in the payoff matrix, all the players would get exactly the same payoff for any strategy they choose (resulting in a random update rule), leading to a destabilization of the system configuration. Finally, we note that, despite a spatial arrangement is impossible, yet the model is still characterized by some constant (averaged in time) fractions for the strategies of the players in this third phase, as can be clearly seen in the gray regions of the plots in figs. 2, 3, but the collaborative attitude is highly inhibited.

Complex networks. – The SG with S-players can be extended as well on complex networks. In this section, we consider the model on two typical examples of complex networks, namely Watts-Strogatz (WS) [39] and Barabàsi-Albert (BA) [40] graphs.

Watts-Strogatz networks. In the Watts-Strogatz case, we consider two realizations of the networks for different rewiring probabilities ($\theta = 0.1$ and $\theta = 0.5$) in the algorithmic construction of the topology. The global picture for the relationships among the characters resembles the results obtained in the regular graph and the phase space is qualitatively very similar. Figure 5 collects the results for the evolution of the fractions of C-, D- and S-players (suitably averaged in time, see the SM) for varying r and fixed q.

As in the regular case, low values of r lead to the extinction of solitary players. Increasing r, the S-density in the final configuration is nonvanishing and takes over quickly the majority of the nodes (again, when the fraction

[3]We discuss in the SM how to locate these critical r_C values by a standard stability analysis.

[4]The nature of the transition is deepened in the SM.

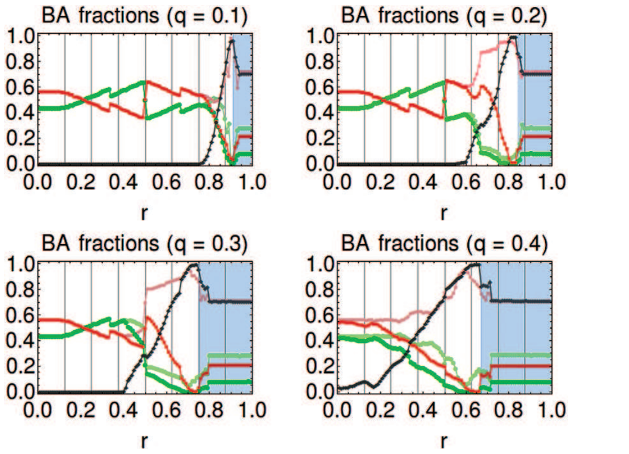

Fig. 6: (Color online) Fractions of cooperators f_C, defectors f_D and solitary players f_S as functions of the parameter r for different choices of q. Green dots denote f_C, red f_D and gray f_S. The shaded regions in the plots correspond to the third phase of the system where no stable spatial arrangement is reached.

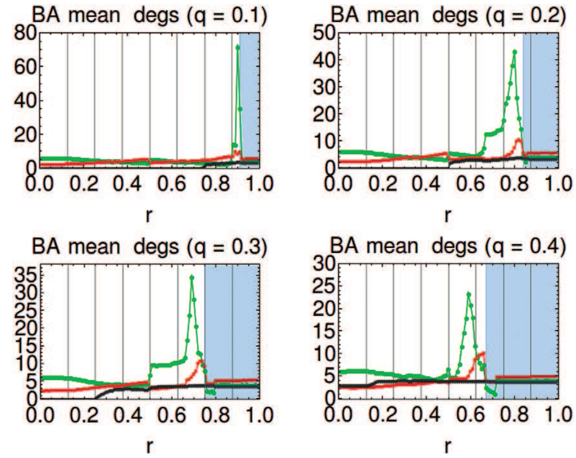

Fig. 7: (Color online) Mean degree of the C, D and S nodes, respectively. Green dots denote the mean degree of the nodes occupied by C-players, red dots are used for D-players and finally gray dots denote S-players. The plots show the mean degree as function of the parameter r for different choices of q. The shaded regions in the plots correspond to the third phase of the system where no stable spatial arrangement is observed.

of the these players approaches 1 the final configuration becomes unstable). Looking at the fractions for $\theta = 0.1$ and $\theta = 0.5$, one can immediately see that the results look extremely similar and qualitatively independent on the rewiring probability.

One may note that, at least in the first phase (without S-players in the final state), one can spot a reminiscence of a steps-like pattern for the C- and D-densities: the WS model is in fact realized starting from a regular graph with a rewiring prescription governed by θ. This procedure preserves the mean degree, and the final degree distribution falls exponentially (in the volume) for large deviations from the mean value. At least for small θ values, this makes the system's dynamics in the WS setting rather close to that on a regular network with the same mean degree for the nodes (essentially assuming $P(k) \to \delta(k-\bar{k})$). Then, by noticing that the evolution of the system remains qualitatively the same even for larger θ values, one could say that the SG game (with or without S-players) is rather insensitive to the *small-world* property of a network (since the strategy choices are almost completely dictated by local interactions among individuals). Increasing the rewiring probability, *i.e.* the randomization of the final graph, has the only effect to slightly and uniformly reduce the convenience of a collaborative behavior.

Barabàsi-Albert networks. The preferential attachment algorithm for the construction of the BA graphs results in a (approximatively for finite size graphs) power law distribution of the nodes. In this case, the local structure of the network is extremely heterogeneous. From the results of the previous sections, one can thus expect that the SG with S-players is characterized by a totally different evolution in the BA setting. This expectation is confirmed by [38], where the authors considered the pure SG with memory in scale-free networks. Without the S-players, the authors [38] found that the cooperative density has a strong, nonmonotonous dependence on the parameter r. In particular, f_C presents peaks at specific values of r, resulting in the nonintuitive picture that a larger payoff for a selfish behavior may lead to an enhancement of the collaboration. As the S-players are included in the game, the above results change drastically: fig. 6 reports the fractions of cooperators f_C, defectors f_D and solitaries f_S as functions of the parameter r, for different choices of q in BA_2 graphs (we denote with BA_2 the Barabàsi-Albert networks constructed adding a node with 2 new links at any step of the growing algorithm). Increasing r for fixed q, most of the nodes opt for the solitary strategy and this in turn strongly inhibits the convenience of the collaborative behavior. Indeed, the fraction of collaborators never grows with r in the presence of the solitary players. This is even more evident by looking at the relative fractions of the C and D behaviors only (excluding the solitary individuals in the normalizations), *i.e* $\bar{f}_{C,D} = \frac{n_{C,D}}{n_C+n_D}$ (reported in lighter colors in fig. 6). The presence of the S-players strongly inhibits the collaboration, the relative fraction $\bar{f}_C$ being constantly lower than $\bar{f}_D$ as the great majority of the players that do not opt for the S-strategy are forced to be defectors. We can better understand the evolution of the dynamics on these networks looking at the mean degree of the nodes occupied by the various players, as shown in fig. 7. For all the values of q, the mean degree of the collaborating nodes has a sharp peak when r grows enough such that the S-players are close to take over the network (*i.e.*, at the onset of the transition to the unstable phase). These plots suggest that, while the solitaries become progressively the largest fraction increasing the payoff for the defection, the collaborative behavior resists only in few nodes with a relatively large degree, where the minority of C-players is segregated and surrounded. The role of high-degree nodes as bastions of least resistance for the cooperative choice is rather unusual. On the other hand, the mean degree of the S-players stays almost constant after their appearance, suggesting that this strategy spreads among the various players as r increases in a rather uniform way.

Conclusions. – By including also *solitary players*, in this paper, we extended the Snowdrift Game with memory to account for a third behavior, whose decisional process relies on global awareness (and not just local knowledge) in a *minority-game attitude*: we studied numerically the phase diagram of this model, namely the evolution of the relative fractions of the three types of players *vs.* the tunable parameters r (the standard cost-to-reward ratio) and q (the solitary payoff). We found robust numerical evidences that, starting by $r = 0$ and $q = 0$, along the r-axes, the evolution of these fractions happens trough discontinuous (first-order–like) phase transitions, while its growth along the q-axes is smooth and coupled to a peak of their fluctuations, as typical in critical (second-order–like) phase transitions. Nevertheless, the model has a third, intrinsically unstable, phase —driven by the global awareness introduced in the payoff matrix— that is reached for high values of (r, q) in a way quite similar to percolation in random graphs.

Concerning the role of the solitary strategy, we found that individuals choosing this behavior show a strong predisposition to group together in clusters for relatively high q, therefore mimicing the tendency of nonconformist people to join together in subcommunities with the same mindset (*e.g.*, Facebook pages, blogs, organizations etc.). However, increasing further the loners' payoff q leads to the unstable phase, in which the gain in the nonconformist behavior is so high that it quickly becomes the majority option, thus turning into a conventional attitude. Then, all of the three strategies becomes equally profitable, resulting in a downsizing of the nonconformist front and making the evolution cyclic. As future outlooks we aim to bypass the pairwise decision rule enlarging the outlined scheme to include also mixed populations [41].

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