

PARAMETER EVALUATION OF A SIMPLE MEAN-FIELD MODEL OF SOCIAL INTERACTION

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The aim of this work is to implement a statistical mechanics theory of social interaction, generalizing econometric discrete choice models. A class of simple mean-field discrete models is introduced and discussed both from the theoretical and phenomenological point of view. We propose a parameter evaluation procedure and test it by fitting the model against three families of data coming from different cases: the estimated interaction parameters are found to have similar positive values, giving a quantitative confirmation of the peer imitation behavior found in social psychology. Furthermore, all the values of the interaction parameters belong to the phase transition regime suggesting its possible role in the study of social systems.

Keywords: Social interactions; empirical model; mean field.

1. Introduction

In recent years there has been an increasing awareness towards the problem of finding a quantitative way to study the role played by human interactions in shaping behavior observed at a population level. Indeed, as early as in the '70s the dramatic consequences of including peer interaction in a mathematical model have been recognized independently by the physics¹² (see also Ref. 14), economics²⁷ and sociology¹⁸ communities. The conclusion reached by all these studies is that mathematical models have the potential of describing several features of social behavior, among which, for example, the sudden shifts often observed in society's aggregate behavior,²¹ and that these are unavoidably linked to the way individual influences each other when deciding how to behave.

The possibility of using such models as a tool of empirical investigation, however, is not found in the scientific literature until the beginning of the present decade⁷: the

reason for this is to be found in the intrinsic difficulty of establishing a methodology of systematic measurement for social features. Confidence that such an aim might be an achievable one has been boosted by the wide consensus gained by econometrics following the Nobel prize awarded in 2000 to economist Daniel McFadden for his work on probabilistic models of discrete choice, and by the increasing interest of policy makers for tools enabling them to cope with the global dimension of today's society.^{19,16} This has led very recently to a number of studies confronting directly the challenge of measuring numerically social interaction for *bottom-up* models, that is, models deriving macroscopic phenomena from assumptions about human behavior at an individual level.^{6,26,24,28}

These works show an interesting interplay of methods coming from econometrics,¹³ statistical physics¹¹ and game theory,²⁰ which reveals a substantial overlap in the basic assumptions driving these three disciplines. It must also be noted that all of these studies rely on a simplifying assumption which considers interaction working on a global uniform scale, that is on a *mean-field* approach. This is due to the inability, stated in Ref. 29, of existing methods to measure social network topological structure in any detail. It is expected that it is only a matter of time before technology allows to overcome this difficulty: in the meanwhile one of the roles of today's empirical studies is to assess how much information can be derived from the existing kind of data such as that coming from surveys, polls and censuses.

This paper considers a mean-field model that highlights the possibility of using the methods of discrete choice analysis to apply a statistical mechanical generalization of the model introduced in Ref. 7. The aim of the paper is twofold. On one hand we are interested in assessing how well the simplest instance of such a model fares when confronted with data, and on the other, we would like to propose a simple procedure of estimation, based on a method developed by Berkson,⁵ that we feel might be very appealing for models at an early stage of development.

2. The Model

While cultural traits are generally modelled with multivalued or continuous variables³ there are situations in which a binary variable is most suitable for the type of available data. Consider indeed a population of individuals facing with a "YES/NO" question, such as choosing between marrying through a religious or a civil ritual, or voting in favor or against the death penalty in a referendum. We index individuals by i , $i = 1, \dots, N$, and assign a numerical value to each individual's choice σ_i in the following way:

$$\sigma_i = \begin{cases} +1 & \text{if } i \text{ says YES,} \\ -1 & \text{if } i \text{ says NO.} \end{cases}$$

Consistently with the widespread use of logit models in econometrics²² and with the statistical mechanics approach to modelling systems of many interacting agents,^{7,8} we assume that the joint probability distribution of these choices may be well

approximated by a Boltzmann–Gibbs distribution corresponding to the following Hamiltonian

$$H_N(\sigma) = - \sum_{i,l=1}^N J_{il}\sigma_i\sigma_l - \sum_{i=1}^N h_i\sigma_i.$$

Heuristically, this distribution favors the agreement of the people’s choices σ_i with some external influence h_i which varies from person to person, and at the same time favors agreement of a couple of people whenever their interaction coefficient J_{il} is positive, whereas favors disagreement whenever J_{il} is negative.

Given the setting, the model consists of two basic steps:

- (i) A parametrization of quantities J_{il} and of h_i .
- (ii) A systematic procedure allowing us to “measure” the parameters characterizing the model, starting from statistical data (such as surveys, polls, etc.).

The parametrization must be chosen to fit as well as possible the data format available, in order to define a model which is able to make good use of the increasing wealth of data available through information technologies.

3. Discrete Choice

Let us first consider our model when it ignores interactions $J_{il} \equiv 0 \forall i, l \in (1, \dots, N)$, that is

$$H_N(\sigma) = - \sum_{i=1}^N h_i\sigma_i.$$

The model shall be applied to data coming from surveys, polls, and censuses, which means that together with the answer to our binary question, we shall have access to information characterizing individuals from a socio-economical point of view. We can formalize such further information by assigning to each person a vector of *socio-economic attributes*

$$a_i = \{a_i^{(1)}, a_i^{(2)}, \dots, a_i^{(k)}\},$$

where

$$a_i^{(1)} = \begin{cases} 1 & \text{for } i \text{ Male,} \\ 0 & \text{for } i \text{ Female,} \end{cases}$$

or

$$a_i^{(2)} = \begin{cases} 1 & \text{for } i \text{ Employee,} \\ 0 & \text{for } i \text{ Self-employed,} \end{cases}$$

etc.

We choose to exploit the supplementary data by assuming that h_i (which is the “field” influencing the choice of i) is a function of the vector of attributes a_i . Since for the sake of simplicity we choose our attributes to be binary variables, the most general form for h_i turns out to be linear

$$h_i = \sum_{j=1}^k \alpha_j a_i^{(j)} + \alpha_0,$$

so that the model’s parameters are given by the components of the vector $\alpha = \{\alpha_0, \alpha_1, \dots, \alpha_k\}$. It is worth pointing out that the parameters $\alpha_j, j = 0, \dots, k$ do not depend on the specific individual i .

This parametrization of h_i correspond to what economists call a *discrete choice* model,²³ and shows a remarkable link between econometrics and statistical mechanics, which is of special interest in view of McFadden’s work concerning this theory and its application to the study of urban transport.

Discrete choice theory states that, when making a choice, each person weights out various factors such as his own gender, age, income, etc., as to maximize in probability the benefit arising from his/her decision. Parameters α tell us the relative weight (i.e. the importance) that the various socio-economic factors have when people are making a decision with respect to our binary question. The parameter α_0 does not multiply any specific attribute, and thus it is a homogeneous influence which is felt by all people in the same way, regardless of their individual characteristics. A discrete choice model is considered good when the parametrized attributes are very suitable for the specific choice, so that the parameter α_0 is found to be small in comparison to the attribute-specific ones.

Elementary statistical mechanics tells us that the probability of an individual i with attributes a_i answering “YES” to our question is the following¹¹:

$$p_i = P(\sigma_i = 1) = \frac{e^{h_i}}{e^{h_i} + e^{-h_i}},$$

$$h_i = \sum_{j=1}^k \alpha_j a_i^{(j)} + \alpha_0.$$

Therefore collecting the choices made by a relevant number of people and keeping track of their socio-economic attributes allows us to use statistics in order to find the value of α for which our distribution best fits the real data. This in turn allows to assess the implications on aggregate behavior if we apply incentives to the population which affect specific attribute, as can be commodity prices in a market situation.

4. Interaction

The kind of model described in the last section has been successfully used by econometrics for the last thirty years,²³ and has opened the way to the quantitative study of social phenomena. Such models, however, only apply to situations where the

functional relation between the people's attributes α and the population's behavior is a smooth one: it is ever more evident, on the other hand, that behavior at a societal level can be marked by sudden jumps.^{24,26,21}

There exist many examples from linguistics, economics, and sociology where it has been observed how the global behavior of large groups of people can change in an abrupt manner as a consequence of slight variations in the social structure (such as, for instance, a change in the pronunciation of a language due to a little immigration rate, or as a substantial decrease in crime rates due to seemingly minor action taken by the authorities).^{1,17,21} From a statistical mechanical point of view, these abrupt transition may be considered as phase transitions caused by the interaction between individuals. Indeed, Brock and Durlauf have shown in Ref. 7 how discrete choice can be extended to the case where a global mean-field interaction is present (providing an interesting mapping to the Curie–Weiss theory¹¹), thus further highlighting the close relation existing between the econometric and the statistical mechanical approaches to problems concerning many agents.

We then go back to studying the general interacting model

$$H_N(\sigma) = - \sum_{i,l=1}^N J_{il}\sigma_i\sigma_l - \sum_{i=1}^N h_i\sigma_i, \tag{4.1}$$

while keeping

$$h_i = \sum_{j=1}^k \alpha_j a_i^{(j)} + \alpha_0.$$

We now need to find a suitable parametrization for the interaction coefficients J_{il} . Since each person is characterized by k binary socio-economic attributes, the population can be naturally partitioned into 2^k subgroups, which for convenience we take of equal size: this leads us to consider a mean-field kind of interaction, where coefficients J_{il} depend explicitly on such a partition. We can express this as follows:

$$J_{il} = \frac{1}{2^k N} J_{gg'}, \quad \text{if } i \in g \text{ and } l \in g',$$

which in turn allows us to rewrite (4.1) as

$$H_N(\sigma) = - \frac{N}{2^k} \left(\sum_{g,g'=1}^{2^k} J_{gg'} m_g m_{g'} + \sum_{g=1}^{2^k} h_g m_g \right),$$

where m_g is the average opinion of group g :

$$m_g = \frac{1}{2^k N} \sum_{i=(g-1)N/2^k+1}^{gN/2^k} \sigma_i.$$

In Ref. 15 the case $k = 1$ of this model was considered: the model's thermodynamic limit was proved, and it was given a rigorous derivation of the model's solution, as

well as an analysis of some analytic properties. In particular, it was shown that the model factorizes completely, so that all the information about the model consists of the equilibrium states:

$$\bar{m}_1 = \tanh(J_{11}\bar{m}_1 + J_{12}\bar{m}_2 + h_1), \quad (4.2)$$

$$\bar{m}_2 = \tanh(J_{21}\bar{m}_1 + J_{22}\bar{m}_2 + h_2). \quad (4.3)$$

This allows us, in particular, to write the probability of i choosing YES in a closed form, similar to the non-interacting one:

$$p_i = P(\sigma_i = 1) = \frac{e^{U_g}}{e^{U_g} + e^{-U_g}}, \quad (4.4)$$

where

$$U_g = \sum_{g'=1}^2 J_{g,g'} \bar{m}_{g'} + h_g.$$

This is the basic tool needed to estimate the model starting from real data. We describe the estimation procedure in the next section.

5. Estimation

We have seen that according to the model, an individual i belonging to group g has probability of choosing “YES” equal to

$$p_i = \frac{e^{U_g}}{e^{U_g} + e^{-U_g}},$$

where

$$U_g = \sum_{g'} J_{g,g'} \bar{m}_{g'} + h_g.$$

The standard approach of statistical estimation for discrete choice models is to maximize the probability of observing a sample of data with respect to the parameters of the model (see e.g. Ref. 4). This is done by maximizing the likelihood function¹³

$$L = \prod_i p_i,$$

with respect to the model's parameters, which in our case consist of the interaction matrix J and the vector α .

Our model, however, is such that p_i is a function of the equilibrium states m_g , which in turn are *discontinuous* functions of the model's parameters. This problem takes away much of the appeal of the maximum likelihood procedure, and calls for a more feasible alternative.

The natural alternative to maximum likelihood for problems of model regression is given by the least squares method,¹³ which simply minimizes the squared norm of the difference between observed quantities, and the model's prediction. Since in our case the observed quantities are the empirical average opinions \tilde{m}_g , we need to find the parameter values which minimize

$$\sum_g (\tilde{m}_g - \tanh U_g)^2, \quad (5.1)$$

which in our case correspond to satisfying as closely as possible the state equations (4.2) in squared norm. This, however, is still computationally cumbersome due to the nonlinearity of the function $\tanh(U_j)$. This problem has already been encountered by Berkson back in the '50s, when developing a statistical methodology for bioassay⁵: this is an interesting point, since this stimulus-response kind of experiment bears a close analogy to the natural kind of applications for a model of social behavior, such as linking stimula given by incentive through policy and media, to behavioral responses on part of a population. Furthermore the same approach is used by statistical mechanics, for example within the problem of finding the proper order parameter for a given Hamiltonian.²

The key observation in Berkson's paper is that, since U_g is a linear function of the model's parameters, and the function $\tanh(x)$ is invertible, a viable modification to least squares is given by minimizing the following quantity, instead:

$$\sum_g (\operatorname{arctanh} \tilde{m}_g - U_g)^2. \quad (5.2)$$

This reduces the problem to a linear least squares problem which can be handled with standard statistical software, and Berkson finds an excellent numerical agreement between this method and the standard least squares procedure.

There are nevertheless a number of issues with Berkson's approach, which are analyzed in Ref. 4, p. 96. All the problems arising can be traced to the fact that to build (5.2), we are collecting the individual observations into subgroups, each of average opinion m_g . The problem is well exemplified by the case in which a subgroup has average opinion $m_g \equiv \pm 1$: in this case $\operatorname{arctanh} m_g = -\infty$, and the method breaks down. However, the event $m_g \equiv \pm 1$ has a vanishing probability when the size of the groups increases, so that the method behaves properly for large enough samples.

The proposed measurement technique is best elucidated by showing a few simple concrete examples, which we do in the next section.

6. Case Studies

We shall carry out the estimation program for three real situations which correspond to a very simple case of our model. The data was obtained from periodical censuses carried out by Istat^a: since census data concerns events which are recorded in official

^aThe Italian National Institute of Statistics.

documents, for a large number of people, we find it to be an ideal testing ground for our model.

For the sake of simplicity, individuals are described by a single binary attribute characterizing their place of residence (either Northern or Southern Italy) and we choose, among the several possible case studies, the ones for which choices are likely to involve peer interaction in a major way.

The first phenomenon we choose to study concerns the share of people who choose to marry through a religious ritual, rather than through a civil one. The second case deals with divorces: here individuals are faced with the choice of a consensual/non-consensual divorce. The last test we perform regards the study of suicidal tendencies, in particular the mode of execution.

6.1. *Civil versus religious marriage in Italy, 2000–2006*

To address this first task we use data from the annual report on the institution of marriage compiled by Istat in the seven years going from 2000 to 2006. The reason for choosing this specific social question is both a methodological and a conceptual one.

Firstly, we are motivated by the exceptional quality of the data available in this case, since it is a census which concerns a population of more than 250,000 people per year, for seven years. This allows us some leeway from the possible issues regarding the sample size, such as the one highlighted in the last section. And just as importantly the availability of a time series of data measured at even times also allows to check the consistency of the data as well as the stability of the phenomenon. Secondly, marriage is probably one of the few matters where a great number of individuals makes a genuine choice concerning their life that gets recorded in an official document, as opposed to what happens, for example, in the case of opinion polls.

We choose to study the data with one of the simplest forms of the model: individuals are divided according to only a binary attribute $a^{(1)}$, which takes value 1 for people from Northern Italy, and 0 for people from Southern Italy. In the formalism of Sec. 2, therefore, the model is defined by the Hamiltonian

$$H_N(\sigma) = -\frac{N}{2}(J_{11}m_1^2 + (J_{12} + J_{21})m_1m_2 + J_{22}m_2^2 + h_1m_1 + h_2m_2),$$

$$h_i = \alpha_1 a_i^{(1)} + \alpha_0,$$

and the state equations to be used for Berkson's statistical procedure are given by (4.4).

Table 1 shows the time evolution of the share of men choosing to marry through a religious ritual: the population is divided into two geographical classes. The first thing worth noticing is that these percentages show a remarkable stability over the seven-year period: this confirms how, though arising from choices made by distinct individuals, who bear extremely different personal motivations, the aggregate behavior can be seen as an observable feature characterizing society as a whole.

In order to apply Berkson's method of estimation, we gather the data into periods of four years, starting with 2000–2003, then 2001–2004, etc. Now, if we label the

Table 1. Percentage of religious marriages, by year and geographical region.

Region	% of religious marriages, by year						
	2000	2001	2002	2003	2004	2005	2006
Northern Italy	68.35	64.98	61.97	54.64	57.91	55.95	54.64
Southern Italy	81.83	80.08	79.32	75.46	76.81	76.52	75.46

share of men in group g choosing the religious ritual in a specific year (say in 2000) by m_g^{2000} , we have that the quantity that ought to be minimized in order to estimate the model’s parameters for the first period is the following, which we label X^2 :

$$X^2 = \sum_{\text{year}=2000}^{2003} \sum_{g=1}^2 (\operatorname{arctanh} m_g^{\text{year}} - U_g^{\text{year}})^2,$$

$$U_g^{\text{year}} = \sum_{g'=1}^2 J_{g,g'} m_{g'}^{\text{year}} + h_g,$$

$$h_g = \alpha_1 a_g^{(1)} + \alpha_0.$$

The results of the estimation for the four periods are shown in Table 2, whereas Table 3 shows the corresponding estimation for a discrete choice model which does not take into account interaction.

6.2. Divorces in Italy, 2000–2005

The second case study uses data from the annual report on divorce trends compiled by Istat in the six years going from 2000 to 2005. The data shows how divorcing couples chose between a consensual and a non-consensual divorce in Northern and

Table 2. Religious vs. civil marriages: estimation for the interacting model.

Parameter	4-year period			
	2000–2003	2001–2004	2002–2005	2003–2006
α_0	-0.10 ± 0.42	-0.16 ± 0.15	-0.18 ± 0.10	-0.13 ± 0.01
α_1	0.20 ± 0.59	0.20 ± 0.22	0.16 ± 0.14	0.14 ± 0.01
J_1	1.16 ± 0.41	1.09 ± 0.16	1.01 ± 0.11	1.02 ± 0.01
J_2	1.29 ± 0.89	1.40 ± 0.33	1.45 ± 0.21	1.36 ± 0.01
J_{12}	-0.21 ± 0.89	-0.10 ± 0.33	0.03 ± 0.21	-0.01 ± 0.01
J_{21}	0.09 ± 0.41	0.02 ± 0.16	-0.01 ± 0.11	0.01 ± 0.01

Table 3. Religious vs. civil marriages: estimation for the non-interacting model.

Parameter	4-year period			
	2000–2003	2001–2004	2002–2005	2003–2006
α_0	0.67 ± 0.15	0.63 ± 0.03	0.61 ± 0.06	0.58 ± 0.03
α_1	-0.41 ± 0.1	-0.43 ± 0.04	-0.45 ± 0.08	-0.46 ± 0.04

Table 4. Percentage of consensual divorces, by year and geographical region.

Region	% of consensual divorces, by year					
	2000	2001	2002	2003	2004	2005
Northern Italy	75.06	80.75	81.32	81.62	81.55	81.58
Southern Italy	58.83	72.80	71.80	72.61	72.76	72.08

Southern Italy. As shown in Table 4 here too, when looking at the ratio among consensual versus the total divorces, the data show a remarkable stability.

Again we gather the data into periods of four years and Table 5 presents the estimation of our model's parameters for the whole available period, while in Table 6 we show the corresponding fit by the non-interacting discrete choice model. We notice that the estimated parameters have some analogies with the preceding case study in that here too the cross-interactions J_{12}, J_{21} are statistically close to zero whereas the diagonal values J_{11}, J_{22} are both greater than one suggesting an interaction scenario which is due to multiple equilibria.¹⁵ Furthermore, in both cases the attribute-specific parameter α_1 is larger than the generic parameter α_0 in the interacting model (Tables 2 and 5), as opposed to what we see in the non-interacting case (Tables 3 and 6): this suggests that by accounting for interaction we might be able to better evaluate the role played by socio-economic attributes.

6.3. Suicidal tendencies in Italy, 2000–2007

The last case study deals with suicidal tendencies in Italy, following the annual report compiled by Istat in the six years from 2000 to 2007, and we use the same geographical attribute used for the former two studies.

Table 5. Consensual vs. non-consensual divorces: estimation for the interacting model.

Parameter	4-year period		
	2000–2003	2001–2004	2002–2005
α_0	0.02 ± 0.06	-0.08 ± 0.01	-0.07 ± 0.01
α_1	-0.25 ± 0.08	-0.22 ± 0.01	-0.23 ± 0.01
J_1	1.59 ± 0.14	1.64 ± 0.01	1.66 ± 0.01
J_2	1.16 ± 0.06	1.25 ± 0.01	1.25 ± 0.01
J_{12}	-0.05 ± 0.06	0.01 ± 0.01	0.00 ± 0.01
J_{21}	-0.08 ± 0.14	0.00 ± 0.01	-0.01 ± 0.01

Table 6. Consensual vs. non-consensual divorces: estimation for the non-interacting model.

Parameter	4-year period		
	2000–2003	2001–2004	2002–2005
α_0	0.41 ± 0.13	0.48 ± 0.01	0.480046 ± 0.01
α_1	0.28 ± 0.18	0.25 ± 0.02	0.261956 ± 0.01

Table 7. Percentage of suicides with hanging as mode of execution, by year and geographical region.

Region	% of suicides by hanging, year							
	2000	2001	2002	2003	2004	2005	2006	2007
Northern Italy	34.17	37.02	35.83	34.58	35.21	36.23	33.57	38.08
Southern Italy	37.10	37.40	37.34	38.54	34.71	38.90	40.63	36.66

The data in Table 7 shows the percentage of deaths due to hanging as a mode of execution. The topic of suicide is of particular relevance to sociology: indeed, the very first systematic quantitative treatise in the social sciences was carried out by Émile Durkheim,⁹ a founding father of the subject, who was puzzled by how a phenomenon as unnatural as suicide could arise with the astonishing regularity that he found. Such a regularity as even been dimmed “sociology’s one law”,²⁵ and there is hope that the connection to statistical mechanics might eventually shed light on the origin of such a law.

Mirroring the two previous case studies, we present the time series in Table 7, whereas Table 8 shows the estimation results for the interacting model, and Table 9, shows the estimation results for the non-interacting discrete choice model. Once again, the data agrees with the analogies found for the two previous case studies.

7. Comments

We introduced a class of simple mean-field models of choice in the presence of social interaction, which generalizes the model studied in Ref. 7. After showing how our model reduces to a standard discrete choice model when we neglect interaction, we analyzed the simplest kind of interaction (by accounting for only one social

Table 8. Suicidal tendencies: estimation for the interacting model.

Parameter	4-year period				
	2000–2003	2001–2004	2002–2005	2003–2006	2004–2007
α_0	0.01 ± 0	0.02 ± 0.01	0.01 ± 0.01	0.02 ± 0.01	0.02 ± 0.01
α_1	0.01 ± 0.01	0.00 ± 0.01	0.00 ± 0.01	0.00 ± 0.01	0.00 ± 0.01
J_1	1.09 ± 0.01	1.09 ± 0.01	1.09 ± 0.02	1.10 ± 0.03	1.09 ± 0.01
J_2	1.06 ± 0.01	1.08 ± 0.01	1.08 ± 0.01	1.07 ± 0.01	1.07 ± 0.01
J_{12}	0 ± 0.01	0.00 ± 0.01	0.00 ± 0.01	0.00 ± 0.01	0.00 ± 0.01
J_{21}	0 ± 0.01	0.01 ± 0.01	0.00 ± 0.02	0.01 ± 0.03	0.01 ± 0.01

Table 9. Suicidal tendencies: estimation for the non-interacting model.

Parameter	4-year period				
	2000–2003	2001–2004	2002–2005	2003–2006	2004–2007
α_0	-0.25 ± 0.02	-0.27 ± 0.03	-0.26 ± 0.03	-0.24 ± 0.04	-0.25 ± 0.05
α_1	-0.05 ± 0.03	-0.03 ± 0.04	-0.04 ± 0.04	-0.07 ± 0.06	-0.04 ± 0.07

attribute): in this case the model reduces to a well-known bipartite model, whose thermodynamic limit as well as multiple equilibria have already been shown to exist.¹⁵

In order to test our model we considered three case studies, concerning relevant social phenomena such as marriage, divorce, and suicide, and we found that Berkson's method of estimation⁵ provides a valuable statistical tool, alternative to the more typical maximum likelihood procedure used in econometrics, which is not suitable for our model due to discontinuities arising in its probability structure.

This paper aims to suggest the outline of a method that can be used to study more specific situations, where individuals may be modelled in a more precise way, by assigning more socio-economic attributes to them. In this simple case we were able to find consistencies in the interaction parameters regarding different topics for the same population. Furthermore, the parameters values were found to be in a regime characterized by multiple equilibria, which suggests the possibility that a refinement of this study will eventually lead to the capability of predicting abrupt transitions at a societal level.

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References

1. P. Ball, *Critical Mass* (Arrow books, 2004).
2. A. Barra, The mean-field Ising model through interpolating techniques, *J. Statist. Phys.* **145** (2008) 234–261.
3. N. Bellomo and M. Delitala, From the mathematical kinetic, and stochastic game theory to modeling mutations, onset, progression and immune competition of cancer cells, *Phys. Life. Rev.* **5** (2008) 183–206.
4. M. Ben-Akiva and S. R. Lerman, *Discrete Choice Analysis* (The MIT Press, 1985).
5. J. Berkson, A statistically precise and relatively simple method of estimating the bioassay with quantal response, based on the logistic function, *J. Amer. Statist. Assoc.* **48** (1953) 565–599.
6. C. Borghesi and J. P. Bouchaud, Of songs and men: A model for multiple choice with herding, *Quality and Quantity* **41** (2007) 557–568.
7. W. Brock and S. Durlauf, Discrete choice with social interactions, *Rev. Econ. Stud.* **68** (2001) 235–260.
8. D. Chandler, *Introduction to Modern Statistical Mechanics* (Oxford Univ. Press, 1987).
9. E. Durkheim, *Le Suicide. Étude de Sociologie* (Paris Alcan, 1897).
10. R. Cont and M. Lowe, Social distance, heterogeneity and social interaction, Centre des Mathématiques Appliquées, École Polytechnique R.I. No. **505** (2003).
11. R. S. Ellis, *Large Deviations and Statistical Mechanics* (Springer, 1985).

12. H. Föllmer, Random economies with many interacting agents, *J. Math. Econ.* **1** (1973) 51–62.
13. D. A. Hensher, J. M. Rose and W. H. Greene, *Applied Choice Analysis: A Primer* (Cambridge Univ. Press, 2005).
14. S. Galam and S. Moscovici, Towards a theory of collective phenomena: Consensus and attitude changes in groups, *Euro. J. Social Psychol.* **21** (1991) 49–74.
15. I. Gallo and P. Contucci, Bipartite mean-field spin systems. Existence and solution, *MPEJ* **14** (2008) 1–21.
16. F. Gallo, P. Contucci, A. Coutts and I. Gallo, Tackling climate change through energy efficiency: Mathematical models for evidence-based public policy recommendations, arXiv:0804.3319.
17. M. Gladwell, *The Tipping Point* (Little, Brown and Company 2000).
18. M. Granovetter, Threshold models of collective behavior, *Amer. J. Sociol.* **83** (1978) 1420–1443.
19. D. Halpern, C. Bates, G. Mulgan and S. Aldridge, Personal responsibility and changing behavior: The state of knowledge and its implications for public policy, Cabinet Office, S Unit, G Britain (2004).
20. B. Kenneth, *Game Theory and the Social Contract* (MIT Press, 1998).
21. T. Kuran, Now out of never, *World Politics* (1991).
22. R. Luce, *Individual Choice Behavior: A Theoretical Analysis* (John Wiley & Sons, 1959).
23. D. McFadden, Economic choices, *Amer. Econ. Rev.* **91** (2001) 351–378.
24. Q. Michard and J. P. Bouchaud, Theory of collective opinion shifts: From smooth trends to abrupt swings, *Euro. Phys. J. B* **47** (2001) 151–159.
25. W. Pope and N. Danigelis, Sociology's one law, social forces (1981).
26. M. J. Salganik, P. S. Dodds and D. J. Watts, Experimental study of inequality and unpredictability in an artificial cultural market, *Science* **311** (2006) 854–856.
27. T. Schelling, *Micromotives and Macrobehavior* (W. W. Norton & Co., 1978).
28. A. R. Soetevent and P. Kooreman, A discrete-choice model with social interactions: With an application to high school teen behavior, *J. Appl. Econ.* **22** (2007) 599–624.
29. D. J. Watts and P. S. Dodds, Influentials, networks and public opinion formation, *J. Consumer Res.* **34** (2007) 441–458.