# Honoring Joel



Abstract from a special volume of the Journal of Statistical Physics

#### Preface

The following papers are dedicated to honoring Joel Lebowitz on the occasion of his ninetieth birthday and his retirement as editor-in-chief of the Journal of Statistical Physics (JSP). These contributions are written by a subset of his many friends, co-authors and admirers.

JSP was founded in 1969 by Howard Reiss, then a professor of chemistry at UCLA. Following the pioneering example of Lars Onsager, many chemists did research in statistical physics, rather than just in the more traditional field of thermodynamics. In the description of the board of editors of that time one reads: "Professor Joel L. Lebowitz, Statistical Mechanics of equilibrium and nonequilibrium, biomathematics, biophysics" – an impressive range of interests and competences.

In 1975, Joel was appointed editor-in-chief of JSP. Browsing through the volumes of this journal produced under Joel's leadership, one observes a substantial change in its scientific orientation. Joel was an active player and catalyser in an area of statistical physics that had emerged in the 60's and was now featuring spectacular progress: rigorous statistical mechanics. Papers published in JSP no longer came out of only physics or chemistry departments but were often written in mathematics departments. Besides statistical mechanics, Joel showed a very lively interest in several other fields, such as deterministic chaos, which then got featured in JSP. As one example, the late Mitchell Feigenbaum's seminal analysis of universality in the period-doubling bifurcations exhibited by families of maps from the unit interval to itself was published in JSP.

Joel made use of JSP to announce the programs of his legendary Statistical Mechanics Conferences. The first one to be announced there was number 34. It took place in December 1975 when Joel was still a professor at Yeshiva University. In December of this year the 123rd conference is scheduled to take place. In 1977, Joel moved to Rutgers University, where he holds the prestigious George William Hill Professorship of Mathematics and Physics.

The editorial board of JSP has traditionally been rather large, on the order of thirty members. It has been rejuvenated periodically. Its members have been tasked with attracting excellent papers to the journal and assisting in the reviewing of papers submitted for publication. Joel handled every single paper sent to the journal. Years ago, one would see stacks of brownish folders filled with papers considered for publication in JSP on his desk, in his car and at home. Nowadays, such stacks have become virtual – thanks to the 'Editorial Manager'. During his forty-three years of service as editor-in-chief, over 10,000 papers submitted to JSP have gone through Joel's hands, an extraordinary accomplishment! Besides taking care of JSP, Joel has also edited, jointly with C. Domb, the series of review volumes entitled Phase Transitions and Critical Phenomena.

Joel has always had a brimming bag of open problems. In his scientific work he has emphasized the synergy between deep questions in science and mathematics. Mentoring and stimulating numerous students, postdoctoral researchers, and colleagues, his style has been to call attention to specific yet far reaching questions, formulated in the context of mathematical models. Forever fascinated with the origin of irreversibility in deterministic dynamics and with the meaning of entropy, Joel would often see far past technicalities, guiding collaborators in developing the details of an argument, all while keeping an eye on the essence which they reveal. Not surprisingly, he has co-authored hundreds of scientific papers spanning all aspects of statistical physics, many of which have become classics.

Beyond being a role model as a scientist and editor, Joel has been setting an example in his support of human rights, urging all to adopt it as both our individual and professional calling. He has had a life-long commitment to helping colleagues all over the world who were or are deprived of basic human rights and who are victims of persecution. The engagement of people like Joel is clearly crucial even if their efforts often take a long time to bear fruit.

We join David Ruelle, who said in his laudation when Joel was awarded the Henri Poincaré Prize: "I think we all recognise that Joel Lebowitz, as a scientist and as a man, has done a lot to make the world a better place to live. And for that we deeply thank him."

With great admiration, the co-editors of this special volume in Joel's honor:

Michael Aizenman Ivan Corwin Jürg Fröhlich Giovanni Gallavotti Shelly Goldstein Herbert Spohn

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#### Glassy states: the free Ising model on a tree

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**Abstract:** We consider the ferromagnetic Ising model on the Cayley tree and we investigate the decomposition of the free state into extremal states below the spin glass temperature. We show that this decomposition has uncountably many components. The tail observable showing that the free state is not extremal is related to the Edwards-Anderson parameter, measuring the variance of the (random) magnetization obtained from drawing boundary conditions from the free state.

# Nonlinear fluctuating hydrodynamics for the classical XXZ spin chain

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We thank Joel Lebowitz for being the very special person that he is: always a pleasure to be with and to collaborate with, and always so helpful to so many people in so many ways.

**Abstract:** Using the framework of nonlinear fluctuating hydrodynamics (NFH), we examine equilibrium spatio-temporal correlations in classical ferromagnetic spin chains with nearest neighbor interactions. In particular, we consider the classical XXZ-Heisenberg spin chain (also known as Lattice Landau Lifshitz or LLL model) evolving deterministically and chaotically via Hamiltonian dynamics, for which energy and z-magnetization are the only locally conserved fields. For the easy-plane case, this system has a low-temperature regime in which the difference between neighboring spin's angular orientations in the XY plane is an almost conserved field. According to the predictions of NFH, the dynamic correlations in this regime exhibit a heat peak and propagating sound peaks, all with anomalous broadening. We present a detailed molecular dynamics test of these predictions and find a reasonably accurate verification. We find that, in a suitable intermediate temperature regime, the system shows two sound peaks with Kardar-Parisi-Zhang (KPZ) scaling and a heat peak where the expected anomalous broadening is less clear. In high temperature regimes of both easy plane and easy axis case of LLL, our numerics show clear diffusive spin and energy peaks and absence of any sound modes, as one would expect. We also simulate an integrable version of the XXZ-model, for which the ballistic component instead moves with a broad range of speeds rather than being concentrated in narrower peaks around the sound speed.

# Divergence of the effective mass of a polaron in the strong coupling limit

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Dedicated to Joel Lebowitz in appreciation of his multifaceted contributions to statistical mechanics

**Abstract:** We consider the Fröhlich model of a polaron, and show that its effective mass diverges in the strong coupling limit.

### Bohmian Trajectories for Hamiltonians with Interior-Boundary Conditions

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Abstract: Recently, there has been progress in developing interior-boundary conditions (IBCs) as a technique of avoiding the problem of ultraviolet divergence in nonrelativistic quantum field theories while treating space as a continuum and electrons as point particles. An IBC can be expressed in the particle-position representation of a Fock vector as a condition on the values  $\psi$  on the set of collision configurations, and the corresponding Hamiltonian is defined on a domain of vectors satisfying this condition. We describe here how Bohmian mechanics can be extended to this type of Hamiltonian. In fact, part of the development of IBCs was inspired by the Bohmian picture. Particle creation and annihilation correspond to jumps in configuration space; the annihilation is deterministic and occurs when two particles (of the appropriate species) meet, whereas the creation is stochastic and occurs at a rate dictated by the demand for the equivariance of the  $|\psi|^2$  distribution, time reversal symmetry, and the Markov property. The process is closely related to processes known as Bell-type quantum field theories.

# Schrödinger's paradox and proofs of nonlocality using only perfect correlations

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Dedicated to our mentor, Joel L. Lebowitz, a master of statistical physics who, when it came to foundational issues of quantum mechanics, was always willing to listen.

**Abstract:** We discuss proofs of nonlocality based on a generalization by Erwin Schrödinger of the argument of Einstein, Podolsky and Rosen. These proofs do not appeal in any way to Bell's inequalities. Indeed, one striking feature of the proofs is that they can be used to establish nonlocality solely on the basis of suitably robust perfect correlations. First we explain that Schrödinger's argument shows that locality and the perfect correlations between measurements of observables on spatially separated systems implies the existence of a non-contextual value-map for quantum observables; noncontextual means that the observable has a particular value before its measurement, for any given quantum system, and that any experiment "measuring this observable" will reveal that value. Then, we establish the impossibility of a non-contextual valuemap for quantum observables without invoking any further quantum predictions. Combining this with SchrÄűdinger's argument implies nonlocality. Finally, we illustrate how Bohmian mechanics is compatible with the impossibility of a non-contextual valuemap.

#### Generalized Gibbs ensembles of the classical Toda chain

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It is my great pleasure to dedicate this article to Joel Lebowitz as teacher, as guide to yet unexplored scientific territories, and with gratitude for a lasting friendship.

Abstract: The Toda chain is the prime example of a classical integrable system with strictly local conservation laws. Relying on the Dumitriu-Edelman matrix model, we obtain the generalized free energy of the Toda chain and thereby establish a mapping to the onedimensional log-gas with an interaction strength of order 1/N. The (deterministic) local density of states of the Lax matrix is identified as the object, which should evolve according to generalized hydrodynamics.

### Quantization of the interacting Hall conductivity in the critical regime

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Abstract: The Haldane model is a paradigmatic 2d lattice model exhibiting the integer quantum Hall effect. We consider an interacting version of the model, and prove that for shortrange interactions, smaller than the bandwidth, the Hall conductivity is quantized, for all the values of the parameters outside two critical curves, across which the model undergoes a 'topological' phase transition: the Hall coefficient remains integer and constant as long as we continuously deform the parameters without crossing the curves; when this happens, the Hall coefficient jumps abruptly to a different integer. Previous works were limited to the perturbative regime, in which the interaction is much smaller than the bare gap, so they were restricted to regions far from the critical lines. The non-renormalization of the Hall conductivity arises as a consequence of lattice conservation laws and of the regularity properties of the current-current correlations. Our method provides a full construction of the critical curves, which are modified ('dressed') by the electron-electron interaction. The shift of the transition curves manifests itself via apparent infrared divergences in the naive perturbative series, which we resolve via renormalization group methods.

# Quantum motion on shape space and the gauge dependent emergence of dynamics and probability in absolute space and time

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To Joel, an invaluable friend and colleague.

**Abstract:** Relational formulations of classical mechanics and gravity have been developed by Julian Barbour and collaborators. Crucial to these formulations is the notion of shape space. We indicate here that the metric structure of shape space allows one to straightforwardly define a quantum motion, a Bohmian mechanics, on shape space. We show how this motion gives rise to the more or less familiar theory in absolute space and time. We find that free motion on shape space, when lifted to configuration space, becomes an interacting theory. Many different lifts are possible corresponding in fact to different choices of gauges. Taking the laws of Bohmian mechanics on shape space as physically fundamental, we show how the theory can be statistically analyzed by using conditional wave functions, for subsystems of the universe, represented in terms of absolute space and time.

### Non-compact quantum spin chains as integrable stochastic particle processes

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#### To Joel Lebowitz, for his continuous inspiration.

Abstract: In this paper we discuss a family of models of particle and energy diffusion on a one-dimensional lattice, related to those studied previously in [Sasamoto-Wadati], [Barraquand-Corwin] and [Povolotsky] in the context of KPZ universality class. We show that they may be mapped onto an integrable  $\mathfrak{sl}(2)$  Heisenberg spin chain whose Hamiltonian density in the bulk has been already studied in the AdS/CFT and the integrable system literature. Using the quantum inverse scattering method, we study various new aspects, in particular we identify boundary terms, modeling reservoirs in non-equilibrium statistical mechanics models, for which the spin chain (and thus also the stochastic process) continues to be integrable. We also show how the construction of a "dual model" of probability theory is possible and useful. The fluctuating hydrodynamics of our stochastic model corresponds to the semiclassical evolution of a string that derives from correlation functions of local gauge invariant operators of  $\mathcal{N} = 4$  super Yang-Mills theory (SYM), in imaginary-time. As any stochastic system, it has a supersymmetric completion that encodes for the thermal equilibrium theorems: we show that in this case it is equivalent to the  $\mathfrak{sl}(2|1)$  superstring that has been derived directly from  $\mathcal{N} = 4$  SYM.

# A detailed fluctuation theorem for heat fluxes in harmonic networks out of thermal equilibrium

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**Abstract:** We continue the investigation, started in [J. Stat. Phys. 166, 926–1015 (2017)], of a network of harmonic oscillators driven out of thermal equilibrium by heat reservoirs. We study the statistics of the fluctuations of the heat fluxes flowing between the network and the reservoirs in the nonequilibrium steady state and in the large time limit. We prove a large deviation principle for these fluctuations and derive the fluctuation relation satisfied by the associated rate function.

# Tumor growth, R-positivity, multitype branching and quasistationarity

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Abstract: Motivated by tumor growth models we establish conditions for the R-positivity of Markov processes and positive matrices. We then apply them to obtain the asymptotic behavior of the tumors sizes in the supercritical regime.

#### Nonequilibrium and fluctuation relation

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This review is dedicated to Joel Lebowitz to witness my gratitude for his constant guidance and teaching which influenced indelibly my work.

 $\label{eq:Abstract: A review on the fluctuation relation, fluctuation theorem and related topics.$ 

#### Stationary states in infinite volume with non zero current

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DEDICATED TO JOEL LEBOWITZ: To celebrate Joel's birthday a conference (New Trends in Statistical Physics) was held in Siena, 2000. In that conference Ruelle lectured on the existence of stationary measures in infinite systems with non zero currents. After 20 years we have an extension of Ruelle's paper just in time to submit it to this volume celebrating Joel's new anniversary. We hope Joel will like it and make him remember the nice time spent in Siena.

**Abstract:** We study the Ginzburg-Landau stochastic models in infinite domains with some special geometry and prove that without the help of external forces there are stationary measures with non zero current in three or more dimensions.

### The emergence of complexity from a simple model for tissue growth

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This article is dedicated to Joel L. Lebowitz. For generations of scientists, including some of the authors, he has been a role model in humanity, scientific rigor and the joy of new knowledge. In particular, Peter Fratzl owes him more than 35 years of mentoring, support and friendship.

Abstract: The growth of living tissue is known to be modulated by mechanical as well as biochemical signals. We study a simple numerical model where the tissue growth rate depends on a chemical potential describing biochemical and mechanical driving forces in the material. In addition, the growing tissue is able to adhere to a three-dimensional surface and is subjected to surface tension where not adhering. We first show that this model belongs to a wider class of models describing particle growth during phase separation. We then analyse the predicted tissue shapes growing on a solid support corresponding to a cut hollow cylinder, which could be imagined as an idealized description of a broken long bone. We demonstrate the appearance of complex shapes described by Delauney surfaces and reminiscent of the shapes of callus appearing during bone healing. This complexity of shapes arises despite the extreme simplicity of the growth model, as a consequence of the three-dimensional boundary conditions imposed by the solid support.

# Exponential decay of correlations in the 2D random field Ising model

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To Joel, inspiring mentor and friend.

**Abstract:** An extension of the Ising spin configurations to continuous functions is used for an exact representation of the Random Field Ising Model's order parameter in terms of disagreement percolation. This facilitates an extension of the recent analyses of the decay of correlations to positive temperatures, at homogeneous but arbitrarily weak disorder.

# Concentration of multi-overlaps for random dilute ferromagnetic spin models

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Dedicated to Joel Lebowitz.

Abstract: We consider mean field ferromagnetic spin models on dilute random graphs and prove that, with suitable one-body infinitesimal perturbations added to the Hamiltonian, the multi-overlaps concentrate for all temperatures, both with respect to the thermal Gibbs average and the quenched randomness. Results of this nature have been known only for the lowest order overlaps, at high temperature or on the Nishimori line. Here we treat all multi-overlaps by a non-trivial application of Griffiths-Kelly-Sherman correlation inequalities. Our results apply in particular to the pure and mixed p-spin ferromagnets on random dilute Erdoes-Renyi hypergraphs. On physical grounds one expects that multi-overlap concentration is an important ingredient for the validity of the cavity (or replica-symmetric) formula for the pressure of mean field models. However rigorously establishing this formula for the p-spin ferromagnet on a random dilute hypergraph remains an open problem.

#### Limit shape of subpartition-maximizing partitions

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This note is dedicated to Joel Lebowitz in appreciation for his tremendous and ongoing contributions to the world of statistical physics.

Abstract: This is an expository note answering a question posed to us by Richard Stanley, in which we prove a limit shape theorem for partitions of n which maximize the number of subpartitions. The limit shape and the growth rate of the number of subpartitions are explicit. The key ideas are to use large deviations estimates for random walks, together with convex analysis and the Hardy-Ramanujan asymptotics. Our limit shape coincides with Vershik's limit shape for uniform random partitions.

# Interface fluctuations in non equilibrium stationary states: the SOS approximation

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Dedicated to Joel for his important contributions to the theory of phase transition and interfaces.

**Abstract:** We study the 2*d* stationary fluctuations of the interface in the SOS approximation of the non equilibrium stationary state found in [A. De Masi, S. Olla, E. Presutti, JSP, 175, 2019]. We prove that the interface fluctuations are of order  $N^{1/4}$ , N the size of the system. We also prove that the scaling limit is a stationary Ornstein-Uhlenbeck process.

#### Geometric phase curvature statistics

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Abstract: The probability distribution of the magnitude C of the curvature 2-form, that underlies the quantum geometric phase and the reaction force of geometric magnetism, is calculated for an ensemble of three-parameter Hamiltonians represented by the Gaussian unitary ensemble of  $N \times N$  matrices. The distributions are determined analytically: exactly for N = 2 and approximately for  $N \ge 3$ , and compared with simulations. The distributions decay asymptotically as  $1/C^{5/2}$ ; this is a consequence of the codimension of energy-level degeneracies in the ensemble.

#### Split-and-merge in stationary random stirring on lattice torus

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We dedicate this paper to Joel Lebowitz on the occasion of his 90th birthday with deep respect for his scientific and moral accomplishment.

Abstract: We show that in any dimension  $d \ge 1$ , the cycle-length process of stationary random stirring (or, random interchange) on the lattice torus converges to the canonical Markovian split-and-merge process with the invariant (and reversible) measure given by the Poisson-Dirichlet law PD(1), as the size of the system grows to infinity. In the case of transient dimensions,  $d \ge 3$ , the problem is motivated by attempts to understand the onset of long range order in quantum Heisenberg models via random loop representations of the latter.

# Subdiffusion in one-dimensional Hamiltonian chains with sparse interactions

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Dedicated to Joel L. Lebowitz, for being a constant source of inspiration.

**Abstract:** We establish rigorously that transport is slower than diffusive for a class of disordered one-dimensional Hamiltonian chains. This is done by deriving quantitative bounds on the variance in equilibrium of the energy or particle current, as a function of time. The slow transport stems from the presence of rare insulating regions (Griffiths regions). In many-body disordered quantum chains, they correspond to regions of anomalously high disorder, where the system is in a localized phase. In contrast, we deal with quantum and classical disordered chains where the interactions, usually referred to as anharmonic couplings in classical systems, are sparse. The system hosts thus rare regions with no interactions and, since the chain is Anderson localized in the absence of interactions, the non-interacting rare regions are insulating. Part of the mathematical interest of our model is that it is one of the few non-integrable models where the diffusion constant can be rigorously proven not to be infinite.

#### Phase diagram of the quantum random energy model

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**Abstract:** We prove Goldschmidt's formula [Phys. Rev. B 47 (1990) 4858] for the free energy of the quantum random energy model. In particular, we verify the location of the first order and the freezing transition in the phase diagram. The proof is based on a combination of variational methods on the one hand, and bounds on the size of percolation clusters of large-deviation configurations in combination with simple spectral bounds on the hypercube's adjacency matrix on the other hand.

# Improved upper bounds on the asymptotic growth velocity of Eden clusters

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This article is dedicated to Joel Lebowitz for his gentle mentoring of many generations of scientists as the Chief Editor of Journal of Statistical Physics for 40 years.

Abstract: We consider the asymptotic shape of clusters in the Eden model on a ddimensional hypercubical lattice. We discuss two improvements for the well-known upper bound to the growth velocity in different directions by that of the independent branching process (IBP). In the IBP, each cell gives rise to a daughter cell at a neighboring site at a constant rate. In the first improvement, we do not allow such births along the bond connecting the cell to its mother cell. In the second, we iteratively evolve the system by a growth as IBP for a duration  $\Delta t$ , followed by culling process in which if any cell produced a descendant within this interval, who occupies the same site as the cell itself, then the descendant is removed. We study the improvement on the upper bound on the velocity for different dimensions d. The bounds are asymptotically exact in the large-d limit. But in d = 2, the improvement over the IBP approximation is only a few percent.

#### Lyapunov exponents for some isotropic random matrix ensembles

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Abstract: A random matrix with rows distributed as a function of their length is said to be isotropic. When these distributions are Gaussian, beta type I, or beta type II, previous work has, from the viewpoint of integral geometry, obtained the explicit form of the distribution of the determinant. We use these result to evaluate the sum of the Lyapunov spectrum of the corresponding random matrix product, and we further give explicit expressions for the largest Lyapunov exponent. Generalisations to the case of complex or quaternion entries are also given. For standard Gaussian matrices X, the full Lyapunov spectrum for products of random matrices  $I_N + \frac{1}{c}X$  is computed in terms of a generalised hypergeometric function in general, and in terms of a single integral involving a modified Bessel function for the largest Lyapunov exponent.

# Microscopic foundations of kinetic plasma theory: The relativistic Vlasov–Maxwell equations and their radiation-reaction-corrected generalization

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We are honored and pleased to celebrate Joel Lebowitz' 90-th birthday, and his many years of invaluable service to the statistical physics community, with this contribution to the microscopic foundations of relativistic kinetic plasma theory, dedicated to Joel.

**Abstract:** It is argued that the relativistic Vlasov–Maxwell equations of the kinetic theory of plasma approximately describe a relativistic system of N charged point particles interacting with the electromagnetic Maxwell fields in a Bopp–Landé–Thomas–Podolsky (BLTP) vacuum, provided the microscopic dynamics lasts long enough. The purpose of this work is not to supply an entirely rigorous vindication, but to lay down a conceptual road map for the microscopic foundations of the kinetic theory of special-relativistic plasma, and to emphasize that a rigorous derivation seems feasible. Rather than working with a BBGKY-type hierarchy of *n*-point marginal probability measures, the approach proposed in this paper works with the distributional PDE of the actual empirical 1-point measure, which involves the actual empirical 2-point measure in a convolution term. The approximation of the empirical 1-point measure by a continuum density, and of the empirical 2-point measure by a (tensor) product of this continuum density with itself, yields a finite-N Vlasov-like set of kinetic equations which includes radiation-reaction and nontrivial finite-N corrections to the Vlasov–Maxwell-BLTP model. The finite-N corrections formally vanish in a mathematical scaling limit  $N \to \infty$  in which charges  $\propto 1/\sqrt{N}$ . The radiation-reaction term vanishes in this limit, too. The subsequent formal limit sending Bopp's parameter  $\varkappa \to \infty$  yields the Vlasov–Maxwell model.

#### Local minima in disordered mean-field ferromagnets

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Abstract: We consider the complexity of random ferromagnetic landscapes on the hypercube  $\{\pm 1\}^N$  given by Ising models on the complete graph with i.i.d. non-negative edgeweights. This includes, in particular, the case of Bernoulli disorder corresponding to the Ising model on a dense random graph  $\mathcal{G}(N, p)$ . Previous results had shown that, with high probability as  $N \to \infty$ , the gradient search (energy-lowering) algorithm, initialized uniformly at random, converges to one of the homogeneous global minima (all-plus or all-minus). Here, we devise two modified algorithms tailored to explore the landscape at near-zero magnetizations (where the effect of the ferromagnetic drift is minimized). With these, we numerically verify the landscape complexity of random ferromagnets, finding a diverging number of (1spin-flip-stable) local minima as  $N \to \infty$ . We then investigate some of the properties of these local minima (e.g., typical energy and magnetization) and compare to the situation where the edge-weights are drawn from a heavy-tailed distribution.

# Ehrenfests' Wind-Tree model is dynamically richer than the Lorentz gas

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To Joel with love and admiration.

Abstract: We consider a physical Ehrenfests' Wind-Tree model where a moving particle is a hard ball rather than (mathematical) point particle. We demonstrate that a physical periodic Wind-Tree model is dynamically richer than a physical or mathematical periodic Lorentz gas. Namely, the physical Wind-Tree model may have diffusive behavior as the Lorentz gas does, but it has more superdiffusive regimes than the Lorentz gas. The new superdiffusive regime where the diffusion coefficient  $D(t) \sim (\ln t)^2$  of dynamics seems to be never observed before in any model.

#### Annealing and replica-symmetry in Deep Boltzmann Machines

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Dedicated to Joel Lebowitz with profound admiration.

Abstract: In this paper we study the properties of the quenched pressure of a multilayer Sherrington-Kirkpatrick model (a deep Boltzmann Machine in artificial intelligence jargon) whose pairwise interactions are allowed between spins lying in adjacent layers and not inside the same layer nor among layers at distance larger than one. We prove a theorem that bounds the quenched pressure of such a K-layer machine in terms of K Sherrington-Kirkpatrick spin glasses and use it to investigate its annealed region. The replica-symmetric approximation of the quenched pressure is identified and its relation to the annealed one is considered. The paper also presents some observation on the model's architectural structure related to machine learning. Since escaping the annealed region is manda- tory for a meaningful training, by squeezing such region we obtain thermodynamical constraints on the form factors. Remarkably, its optimal escape is achieved by requiring the last layer to scale sub-linearly in the network size.

### Statistics of TASEP with three merging characteristics

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We dedicate this paper to Joel Lebowitz on the occasion of his 90th birthday.

**Abstract:** In this paper we consider the totally asymmetric simple exclusion process, with nonrandom initial condition having three regions of constant densities of particles. From left to right, the densities of the three regions are increasing. Consequently, there are three characteristics which meet, i.e. two shocks merge. We study the particle fluctuations at this merging point and show that they are given by a product of three (properly scaled) GOE Tracy-Widom distribution functions. We work directly in TASEP without relying on the connection to last passage percolation.

### Stationary non equilibrium states in kinetic theory

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Our interest on the subject of non-equilibrium stationary states was stimulated by the interaction with J. L. Lebowitz, by his questions and his stressing the relevance of the problem. Let us spend some words on the beginning of the story. One of us visited Rutgers University in 1987 and the first question of Joel to him was: what about the flow past an obstacle? The papers by Ukai-Asano on the existence of the solution to the Boltzmann equation in the exterior domain were recent and it was natural to ask about the hydrodynamical limit for this problem. It took thirty years to give the answer outlined in this paper and during this time our fruitful collaboration with Lebowitz touched many different subjects, but most of them on the construction of stationary solutions. We take this occasion to thank him for his contagious passion in the scientific research and his warm friendship.

**Abstract:** Stationary non equilibrium solutions to the Boltzmann equation, despite their relevance in applications, are much less studied than time dependent solutions, and no general existence theory is yet available, due to serious technical difficulties. Here we review some results on the construction of stationary non equilibrium solutions, in a general domain in contact with a slightly non-homogeneous thermal reservoir, both for finite and small Knudsen number. We will describe different approaches and different techniques developed. The main focus will be on stationary solutions close to hydrodynamics. In particular, we will give an answer to the longstanding open problem of the rigorous derivation of the steady incompressible Navier-Stokes-Fourier system from the Boltzmann theory, in the presence of a small external force and diffuse boundary condition with small boundary temperature variations.

### Simulations of transport in hard particle systems

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Dedicated to Joel L. Lebowitz to celebrate his key role as leading editor of the Journal of Statistical Physics.

**Abstract:** Hard particle systems are among the most successful, inspiring and prolific models of physics. They contain the essential ingredients to understand a large class of complex phenomena, from phase transitions to glassy dynamics, jamming, or the physics of liquid crystals and granular materials, to mention just a few. As we discuss in this paper, their study also provides crucial insights on the problem of transport out of equilibrium. A main tool in this endeavour are computer simulations of hard particles. Here we review some of our work in this direction, focusing on the hard disks fluid as a model system. In this quest we will address, using extensive numerical simulations, some of the key open problems in the physics of transport, ranging from local equilibrium and Fourier's law to the transition to convective flow in the presence of gravity, the efficiency of boundary dissipation, or the universality of anomalous transport in low dimensions. In particular, we probe numerically the macroscopic local equilibrium hypothesis, which allows to measure the fluid's equation of state in nonequilibrium simulations, uncovering along the way subtle nonlocal corrections to local equilibrium and a remarkable bulk-boundary decoupling phenomenon in fluids out of equilibrium. We further show that the hydrodynamic profiles that a system develops when driven out of equilibrium by an arbitrary temperature gradient obey universal scaling laws, a result that allows the determination of transport coefficients with unprecedented precision and proves that Fourier's law remains valid in highly nonlinear regimes. Switching on a gravity field against the temperature gradient, we investigate numerically the transition to convective flow. We uncover a surprising two-step transition scenario with two different critical thresholds for the hot bath temperature, a first one where convection kicks but gravity hinders heat transport, and a second critical temperature where a percolation transition of streamlines connecting the hot and cold baths triggers efficient convective heat transport. We also address numerically the efficiency of boundary heat baths to dissipate the energy provided by a bulk driving mechanism. As a bonus track, we depart from the hard disks model to study anomalous transport in a related hard-particle system, the 1d diatomic hard-point gas. We show unambiguously that the universality conjectured for anomalous transport in 1d breaks down for this model, calling into question recent theoretical predictions and offering a new perspective on anomalous transport in low dimensions. Our results show how carefully-crafted numerical simulations of simple hard particle systems can lead to unexpected discoveries in the physics of transport, paving the way to further advances in nonequilibrium physics.

### Stress-energy in Liouville conformal field theory

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Abstract: We construct the stress-energy tensor correlation functions in probabilistic Liouville Conformal Field Theory (LCFT) on the two-dimensional sphere  $S^2$  by studying the variation of the LCFT correlation functions with respect to a smooth Riemannian metric on  $S^2$ . In particular we derive conformal Ward identities for these correlation functions. This forms the basis for the construction of a representation of the Virasoro algebra on the canonical Hilbert space of the LCFT.

In [Kupiainen, Rhodes, Vargas, Commun. Math. Phys. 371, 1005–1069 (2019)] the conformal Ward identities were derived for one and two stress-energy tensor insertions using a different definition of the stress-energy tensor and Gaussian integration by parts. By defining the stress-energy correlation functions as functional derivatives of the LCFT correlation functions and using the smoothness of the LCFT correlation functions proven in [Oikarinen, Ann. Henri Poincaré (2019)] allows us to control an arbitrary number of stress-energy tensor insertions needed for representation theory.

### Towards a mathematical model of the brain

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**Abstract:** This article presents an idealized mathematical model of the cerebral cortex, focusing on the dynamical interaction of neurons. The author proposes a network architecture more consistent with neuroanatomy than in previous studies, borrows ideas from nonequilibrium statistical mechanics and calls attention to the fact that the brain is a large and complex dynamical system. The ideas proposed are illustrated with a realistic model of the visual cortex.

### Space, time, categories, mechanics, and consciousness: On Kant and neuroscience

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Dedicated to Joel Lebowitz.

Abstract: Kant (1724-1804) is rarely mentioned in modern neuroscience publications, and equally rarely are insights from the neurosciences discussed in works on Kantian philosophy. In this essay I present a correlation, not a confrontation, between Kant in the 'Critique of Pure Reason' and the neurosciences on space, time, categories, mechanics, and consciousness in order to highlight their mutual importance. My conclusion will be that Kant is still important for modern neuroscience, although historically he lacked all relevant data and concepts about the brain, and that the insights from neuroscience are equally important for philosophy.

### Thermalization of local observables in the $\alpha$ -FPUT chain

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Dedicated to Joel Lebowitz, to thank him for being a constant source of new ideas, for explaining Boltzmann, and for his warmth and kindness.

**Abstract:** Most studies on the problem of equilibration of the Fermi-Pasta-Ulam-Tsingou (FPUT) system have focused on equipartition of energy being attained amongst the normal modes of the corresponding harmonic system. In the present work, we instead discuss the equilibration problem in terms of local variables, and consider initial conditions corresponding to spatially localized energy. We estimate the time-scales for equipartition of space localized degrees of freedom and find significant differences with the times scales observed for normal modes. Measuring thermalization in classical systems necessarily requires some averaging, and this could involve one over initial conditions or over time or spatial averaging. Here we consider averaging over initial conditions chosen from a narrow distribution in phase space. We examine in detail the effect of the width of the initial phase space distribution, and of integrability and chaos, on the time scales for thermalization. We show how thermalization properties of the system, quantified by its equilibration time, defined in this work, can be related to chaos, given by the maximal Lyapunov exponent. Somewhat surprisingly we also find that the ensemble averaging can lead to thermalization of the integrable Toda chain, though on much longer time scales.

### From fluctuating kinetics to fluctuating hydrodynamics: a $\Gamma$ -Convergence of large deviations functionals approach

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**Abstract:** We consider extended slow-fast systems of N interacting diffusions. The typical behavior of the empirical density is described by a nonlinear McKean-Vlasov equation depending on  $\varepsilon$ , the scaling parameter separating the time scale of the slow variable from the time scale of the fast variable. Its atypical behavior is encapsulated in a large N Large Deviation Principle (LDP) with a rate functional  $\mathcal{I}^{\varepsilon}$ . We study the  $\Gamma$ -convergence of  $\mathcal{I}^{\varepsilon}$  as  $\varepsilon \to 0$  and show it converges to the rate functional appearing in the Macroscopic Fluctuations Theory (MFT) for diffusive systems.

### Response theory for static and dynamic solvation of ionic and dipolar solutes in water

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**Abstract:** The response of polar solvents to ions and polar molecules dictates many fundamental molecular processes. To understand such electrostatically-driven solvation processes, one ideally would probe the dielectric response of a solvent to a idealized point test charge or dipole solute, as envisioned in classic continuum treatments of the problem. However, this is difficult in simulations using standard atomically-detailed solvent models with embedded point charges due to possible overlap with the test charge that lead to singular interaction energies. This problem is traditionally avoided for a realistic charged solute by introducing an excluded volume core that shields its embedded point charge or dipole from the charges in the solvent. However, this core introduces additional molecular-scale perturbations of the solvent density that complicate the interpretation of solvent dielectric response. In this work, we avoid these complications through the use of Gaussian-smoothed test charges and dipoles. Gaussian charges and dipoles can be readily inserted anywhere into an atomistic solvent model without encountering infinite energies. If the Gaussian smoothing is on the scale of molecular correlations in the solvent, both the thermodynamic and dynamic solvation response is linear. Using this observation, we construct accurate predictive theories for solvation free energies and solvation dynamics for insertion of Gaussian charges and dipoles in polar solvents and demonstrate the accuracy of the theories for a widely-used model of water. Our results suggest that Gaussian test charge distributions can be used as an informative probe of dielectric response in molecular models, and our theories can be used to analytically predict the largest component of solvation free energies of charged and polar solutes.

### The nonlinear Schrödinger equation on Z and R with bounded initial data: examples and conjectures

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Dedicated to Joel Lebowitz with admiration for his inspiring leadership.

**Abstract:** We study the nonlinear Schrödinger equation (NLS) with bounded initial data which does not vanish at infinity. Examples include periodic, quasi-periodic and random initial data. On the lattice we prove that solutions are polynomially bounded in time for any bounded data. In the continuum, local existence is proved for real analytic data by a Newton iteration scheme. Global existence for NLS with a regularized nonlinearity follows by analyzing a local energy norm.

### Activity induced nematic order in isotropic liquid crystals

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**Abstract:** We use linear stability analysis to show that an isotropic phase of elongated particles with dipolar flow fields can develop nematic order as a result of their activity. We argue that ordering is favoured if the particles are flow-aligning and is strongest if the wavevector of the order perturbation is neither parallel nor perpendicular to the nematic director. Numerical solutions of the hydrodynamic equations of motion of an active nematic confirm the results. The instability is contrasted to the well-known hydrodynamic instability of an ordered active nematic.

## Equivalence of ensembles in Curie-Weiss models using coupling techniques

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Joel L. Lebowitz has been one of the driving forces and main supporters of mathematical statistical physics for over half a century. It is a particular honour and a pleasure to dedicate this, in relation humble, update on foundations of statistical mechanics to him.

Abstract: We consider equivalence of ensembles for two mean field models: the discrete, standard Curie-Weiss model and its continuum version, also called the mean-field spherical model. These systems have two thermodynamically relevant quantities and we consider the three associated standard probability measures: the microcanonical, canonical, and grand canonical ensembles. We prove that there are ranges of parameters where at least two of the ensembles are equivalent. The equivalence is not restricted to proving that the ensembles have the same thermodynamic limit of the specific free energy but we also give classes of observables whose ensemble averages agree in the limit. Moreover, we obtain explicit error estimates for the difference in the ensemble averages. The proof is based on a construction of suitable couplings between the relevant ensemble measures, proving that their Wasserstein fluctuation distance is small enough for the error in the ensemble averages to vanish in the thermodynamic limit. A crucial property for these estimates is permutation invariance of the ensemble measures.

# Slow-to-start traffic model: condensation, saturation and scaling limits

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Abstract: We consider a one-dimensional traffic model with a slow-to-start rule. The initial position of the cars in  $\mathbb{R}$  is a Poisson process of parameter  $\lambda$ . Cars have speed 0 or 1 and travel in the same direction. At time zero the speed of all cars is 0; each car waits an exponential time to switch speed from 0 to 1 and stops when it collides with a stopped car. When the car is no longer blocked, it waits a new exponential time to assume speed one, and so on. We study the emergence of condensation for the saturated regime  $\lambda > 1$  and the critical regime  $\lambda = 1$ , showing that in both regimes all cars collide infinitely often and each car has asymptotic mean velocity  $1/\lambda$ . In the saturated regime the moving cars form a point process whose intensity tends to 1. The remaining cars condensate in a set of points whose intensity tends to zero as  $1/\sqrt{t}$ . We study the scaling limit of the traffic jam evolution in terms of a collection of coalescing Brownian motions.

### Invariance principle for a Potts interface along a wall

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**Abstract:** We consider nearest-neighbor two-dimensional Potts models, with boundary conditions leading to the presence of an interface along the bottom wall of the box. We show that, after a suitable diffusive scaling, the interface weakly converges to the standard Brownian excursion.

### Statistical mechanics of confined polymer networks

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In celebration of the achievements of our dear colleague Joel L. Lebowitz.

Abstract: We show how the theory of the critical behaviour of d-dimensional polymer networks of arbitrary topology can be generalized to the case of networks conned by hyperplanes. This in particular encompasses the case of a single polymer chain in a bridge configuration. We further define multi-bridge networks, where several vertices are in local bridge configurations. We consider all cases of ordinary, mixed and special surface transitions, and polymer chains made of self-avoiding walks, or of mutually-avoiding walks, or at the tricritical  $\Theta$ -point. In the  $\Theta$ -point case, generalising the good-solvent case, we relate the critical exponent for simple bridges, to that of terminally-attached arches, and to the correlation length exponent. For general networks, the explicit expression of configurational exponents then naturally involve bulk and surface exponents for multiple random paths. In two-dimensions, we describe their Euclidean exponents from a unified perspective, using Schramm-Loewner Evolution (SLE) in Liouville quantum gravity (LQG), and the so-called KPZ relation between Euclidean and LQG scaling dimensions. This is done in the case of ordinary, mixed and special surface transitions, and of the  $\Theta$ -point. We provide compelling numerical evidence for some of these results both in two- and three-dimensions.

### Punctures and *p*-spin curves from matrix models

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To our very dear and highly esteemed friend Joel Lebowitz. Se questo è un uomo asked Primo Levi in 1947. No one is more an uomo than Joel.

Abstract: This article investigates the intersection numbers of the moduli space of p-spin curves with the help of matrix models. The explicit integral representations that are derived for the generating functions of these intersection numbers exhibit p Stokes domains, labelled by a "spin"-component l taking values  $l = -1, 0, 1, 2, \ldots, p-2$ . Earlier studies concerned integer values of p, but the present formalism allows one to extend our study to half-integer or negative values of p, which turn out to describe new types of punctures or marked points on the Riemann surface. They fall into two classes : Ramond (l = -1), absent for positive integer p, and Neveu-Schwarz  $(l \neq -1)$ . The intersection numbers of both types are computed from the integral representation of the n-point correlation functions in a large N scaling limit. We also consider a supersymmetric extension of the random matrix formalism to show that it leads naturally to an additional logarithmic potential. Open boundaries on the surface, or admixtures of R and NS punctures, may be handled by this extension.

### A path-integral analysis of interacting Bose gases and loop gases

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Dedicated to Joel L. Lebowitz.

Abstract: We review some recent results on interacting Bose gases in thermal equilibrium. In particular, we study the convergence of the grand-canonical equilibrium states of such gases to their mean-field limits, which are given by the Gibbs measures of classical field theories with quartic Hartree-type self-interaction, and to the Gibbs states of classical gases of point particles. We discuss various open problems and conjectures concerning, e.g., Bose-Einstein condensation, polymers and  $|\phi|^4$ -theory.

### A path-integral analysis of interacting Bose gases and loop gases

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Some Brief History Told By The Second Author: I met Joel for the first time when I came to Bures-sur-Yvette. Very quickly, we became close friends and remain so until now. Soon after we met for the first time, Joel came to Russia where he was introduced to many fellow colleagues who he also became friends with. In fact he visited Russia several times, some of those times with E. Lieb, and visited A.D. Sakharov often. We had many joint papers on different topics. One of our papers was with N. Chernov used new results from ergodic theory to analyze the Lorentz model of a metallic conductor. It was surprising that we could use singular measures (which was also a Sinai-Ruelle-Bowen measure) in the analysis of smooth hyperbolic dynamical systems. In addition, we also published a paper on the trajectories of self-avoiding walks in higher dimensions ( $d \ge 5$ ) [Russian Mathematical Surveys 50.2 (1995)], which is not very well known. It has been a great pleasure to work with Joel due to his great enthusiasm and encyclopedic knowledge of many subjects. He remains a close colleague and a permanent friend and as enthusiastic as ever in his fields of interest.

**Abstract:** In this paper dedicated to Joel Lebowitz, some brief historical facts are mentioned. In addition, some of the known theory concerning the statistical physics of freely fluctuating 2-dimensional crystals subject to a non-linear elastic hamiltonian is described. The particular topics that are discussed include the existence of 2-dimensional crystals in relation to the Hohenberg-Mermin-Wagner theorem, the crumpling transition for freely suspended crystalline membranes and the renormalization of elastic moduli. Although much of what is known has been uncovered since the mid-80s, the topic has become of interest once again due to the discovery of graphene and other 2-dimensional crystals. The field is vast so it is the aim of this note to highlight some of the fundamental features.

## Microscopic irreversibility: looking for a microscopic description of time asymmetry

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This paper is dedicated to Joel Lebowitz on the occasion of his 90th birthday, in celebration of over 64 years of friendship.

Abstract: This paper is an attempt to understand time-reversal asymmetry better by improving the quantitative description of that asymmetry. The aim is not to explain the asymmetry, but to describe it in more detail. Two model systems are considered here; one is the classical Lorentz gas, the other a quantum Lorentz gas. In the classical case, it is argued that the distribution of the directions of motion of particles that are about to hit an obstacle is qualitatively different from the analogous distribution for particles that have just hit the obstacle (an entropy-like property of the velocity distribution function is used to characterize the asymmetry). In the quantum case, a similar distinction is drawn between the density matrix describing particles that have not yet encountered an obstacle and the one describing particles which have hit an obstacle or are in the process of doing so.

## Transmission and navigation on disordered lattice networks, directed spanning forests and scaling limits

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**Abstract:** Stochastic networks based on random point sets as nodes have attracted considerable interest in many applications, particularly in communication networks, including wireless sensor networks, peer-to-peer networks and so on. The study of such networks generally requires the nodes to be independently and uniformly distributed as a Poisson point process. In this work, we venture beyond this standard paradigm and investigate the stochastic geometry of networks obtained from directed spanning forests (DSF) based on randomly perturbed lattices, which have desirable statistical properties as a models of spatially dependent point fields. In the regime of low disorder, we show in 2D and 3D that the DSF almost surely consists of a single tree. In 2D, we further establish that the DSF, as a collection of paths, converges under diffusive scaling to the Brownian web.

### On derivation of the Poisson-Boltzmann equation

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To Joel with friendship and admiration.

**Abstract:** Starting from the microscopic reduced Hartree-Fock equation, we derive the macroscopic linearized Poisson-Boltzmann equation for the electrostatic potential associated with the electron density.

# Thank You Joel & Happy 90th Birthday!

# Best wishes for more to come.

The papers listed in this booklet are scheduled to appear in a dedicated volume of the Journal of Statistical Physics (to appear in the Fall of 2020).