EQ. 1+1 Dim Fourier

$U(x,t)$

$$\frac{\partial U}{\partial t} = K \frac{\partial^2 U}{\partial x^2}$$

Fourier → TEMPERATURA
Fourier → DIFFUSIONE

$$c S \frac{\partial U}{\partial t} = \frac{\partial}{\partial x}\left[K_0 \frac{\partial U}{\partial x}\right] + \cancel{Q(x,t)}$$

1 2 3

$$\boxed{\frac{\partial U}{\partial t} = K \frac{\partial^2 U}{\partial x^2}}$$

PDE

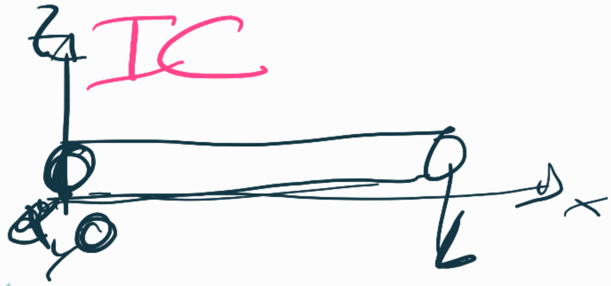
$$\phi\alpha = -K \frac{\partial U}{\partial x}$$

$$\frac{d^2 x}{dt^2} + k^2 x = 0$$

ODE → Cauchy

CONDIZIONI DI BORDO : BC

→ PROFILO INIZIALE : IC




BC :
$$\boxed{\begin{aligned} U(0,t) &= T_1 \\ U(L,t) &= T_2 \end{aligned}}$$

IC : $U(x,t=0) = f(x)$

ALTRO ex

BC $\quad \frac{\partial u}{\partial x}(x=0)=\phi \;\wedge\; \frac{\partial u}{\partial x}(x=L)=\phi$

$$\phi = -K\frac{\partial u}{\partial x}$$

$$\boxed{\cancel{\frac{\partial u}{\partial t}} = K\frac{\partial^2 u}{\partial x^2}}$$

SOLUZIONI STAZIONARIE

↳ $\dot{u}=0 \rightarrow u=u(x)$


ex | sol. staz. di (*) →

$u(x=\phi,t)=T_1 \;\wedge\; u(x=L,t)=T_2$

$$\frac{\partial u}{\partial t}=0 \rightarrow \frac{\partial^2 u}{\partial x^2}=0 \rightarrow \frac{d^2 u}{dx^2}=0$$

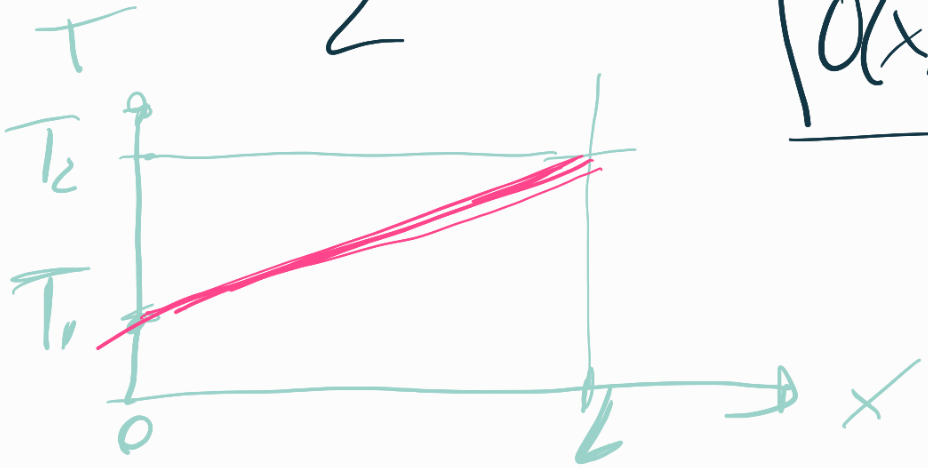
$$\frac{du}{dx}=C_1 \rightarrow \boxed{u(x)=C_1 x + C_2}$$

$U(x=0) = T_1 \rightarrow \boxed{C_2 = T_1}$ *

$U(x=L) = T_2 \rightarrow T_2 = C_1 L + T_1$

$\rightarrow \frac{(T_2 - T_1)}{L} = C_1$ *

$$\boxed{U(x) = \left(\frac{T_2 - T_1}{L}\right) x + T_1}$$




EX

| PDE | BCs | IC |
|---|---|---|
| $\partial_t U = K \partial^2_{xx} U$ | $\partial_x U(x=0) = \phi$, $\partial_x U(x=L) = \phi$ | $U(x, t=0) = f(x)$ |

poiché cerco sol. stazionari

$\rightarrow \partial_t U = 0 \Rightarrow \partial^2_{xx} U = 0 \rightarrow \frac{d^2 U}{dx^2} = 0$

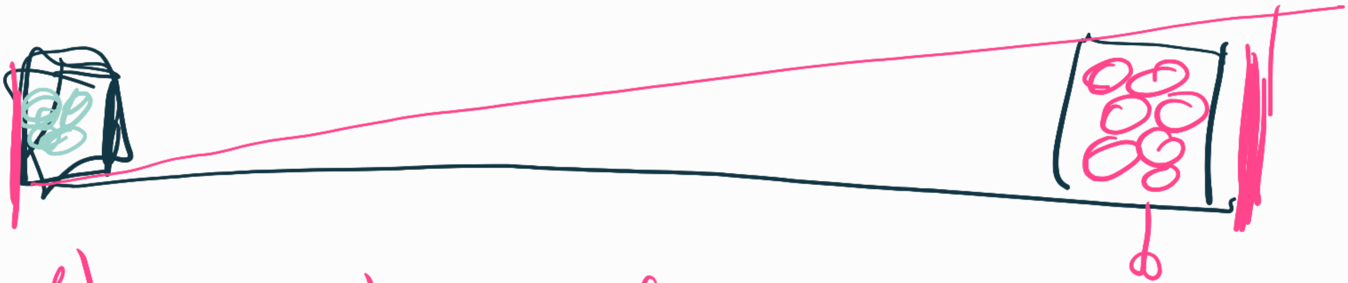
$$\boxed{U(x) = C_1 x + C_2} \rightarrow \frac{dU}{dx} = C_1 \rightarrow C_1 = 0$$

$$\rightarrow \boxed{U(x) = C_2}$$

osservo che, poiché $\boxed{\phi \equiv 0}$, $\rightarrow$ l'energia termica si deve conservare $\boxed{Q = 0}$

$$E = \int_0^L c\rho u\,dx = \begin{cases} t=0: & c\rho\int_0^L f(x)\,dx \\ t \text{ grande}: & c\rho\int_0^L c_2\,dx \end{cases}$$

$$\boxed{c_2 = \frac{1}{L}\int_0^L f(x)\,dx}$$



1) Gradiente Lineare

2) il [illegible] della [illegible]

1-1

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \Rightarrow \frac{\partial u}{\partial t} = k\nabla^2 u$$

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ESERCIZIO Fourier 1D

→ Stazionario

a) $Q=0$, $u(0)=0$, $u(L)=T$

$$\frac{d^2u}{dx^2}=0 \quad u(x) = c_1 x + c_2$$

BC1 → $u(0)=0$ → $\boxed{c_2=0}$

BC2 → $u(L)=T$

$\hookrightarrow T = C_1 L \rightarrow \boxed{C_1 = T/L}$

$\lim_{x \to 0} U(x) = \phi$

Sol. $\rightarrow \boxed{U(x) = \frac{T}{L} x}$

$\lim_{x \to L} U(x) = T$

$\boxed{U(x) = -\frac{T}{L} x + T}$

b) $Q = 0$ $\boxed{\begin{cases} U(\phi) = T \\ U(L) = \phi \end{cases}}$


$U(X) = C_1 X + C_2$ $\boxed{C_1 = -T/L}$

$U(x=0) = T \rightarrow \boxed{C_2 = T}$ $\sim U(x=L) = 0 = C_1 L + T$

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EX $Q = 0, \frac{\partial u}{\partial x}(x=\phi) = \phi$ $U(L) = T$

$\frac{d^2 u}{dx^2} = \phi \rightarrow U(x) = C_1 X + C_2$

$\frac{dU}{dx} = \boxed{C_1 = 0} \rightarrow \boxed{U(x) = C_2}$ $U(x=L) = T$

$\boxed{C_2 = T}$

Caso 2 $\frac{Q}{K_0} = 1, \begin{cases} U(0) = T_1 \\ U(L) = T_2 \end{cases}$

$\boxed{\cancel{c\rho \frac{\partial u}{\partial t}} = K_0 \frac{\partial^2 u}{\partial x^2} + Q\cancel{(x,t)}}$

$\boxed{\frac{d^2 u}{dx^2} = -\frac{Q}{K_0}}$

$$\frac{dU}{dx} = -\frac{Q}{k}x + C = -x + C_1 = \frac{dU}{dx}$$

$\frac{Q}{k} = 1$

$$\boxed{U(x) = -\frac{1}{2}x^2 + C_1 x + C_2}$$

$U(x=0) = T_1$

$U(x=L) = T_2$

$U(x=0) = T_1 = C_2 \rightarrow \boxed{C_2 = T_1}$

$U(x=L) = T_2 = -\frac{1}{2}L^2 + C_1 L + T_1$

$T_2 - T_1 = -\frac{1}{2}L^2 + C_1 L \rightarrow (T_2 - T_1) + \frac{1}{2}L^2 = C_1 L$

$\rightarrow \boxed{C_1 = \frac{1}{L}(T_2 - T_1) + L/2}$ $\boxed{C_2 = T_1}$

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ESERCIZIO sul potenziale (Procedure collaud. con le der. parz.)

Calcolare il potenziale del seguente campo conservativo $\in \mathbb{R}^3$

$\int f(x)\,dx = F(x) + C$

$$F = (y^2z^3 - 6xz^2)\hat{i} + (2xyz^3)\hat{j} + (3xy^2z^2 - 6x^2z)\hat{k}$$

$$F = \nabla U = -\nabla V$$

$\hat{i}$ di $F$ $\quad \hat{j}$

se $\exists U, \exists U : \left(\frac{\partial U}{\partial x} = F_x\right) \wedge \left(\frac{\partial U}{\partial y} = F_y\right) \wedge \left(\frac{\partial U}{\partial z} = F_z\right)$

$$\frac{\partial U}{\partial x} = y^2z^3 - 6xz^2 \qquad (1) \to U = xy^2z^3 - 3x^2z^2 + G(y,z) \quad (4)$$

$$\frac{\partial U}{\partial y} = 2xyz^3 + 0 \qquad (4)\ \frac{\partial U}{\partial y} = 2xyz^3 + 0 + \frac{dG}{dy} = 0 \to G(z)$$

$$\frac{\partial U}{\partial z} = 3xy^2z^2 - 6x^2z + 0 \quad (3)$$

(4bis) $\to U = xy^2z^3 - 3x^2z^2 + G_1(z) \to 3xy^2z^2 - 6x^2z + \frac{dG_1}{dz}$

$$\hookrightarrow \frac{dG}{dz} = 0 \to G = \text{constante} \neq f(x,y,z)$$

$$\boxed{U = xy^2z^3 + 3x^2z^2 + C}, \ \boxed{K \in \mathbb{R}}$$

1) $D=1 \to D=3$ 2) $U(x,t)$

Rate of change of heat energy = Flux of heat energy across the surface + sources $Q$

$$dV = dx\,dy\,dz$$

$$\frac{\partial}{\partial t}\iiint_{\mathbb{R}^3} c\rho\, U(x,t)\, dV = -\oiint_{\Sigma} \vec{\phi}\cdot\hat{n}\, dS + \iiint_{\mathbb{R}^3} Q(x,y,z,t)\, dV$$

$x,y,z,t$

Divergenza di 1 vettore $\Delta$ in $\mathbb{R}^3$


$$\bar{\nabla}\cdot\bar{A} = \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z$$

Teorema della Divergenza, Green Gauss Ostrogradskij, Stokes

$$\boxed{\oint\!\!\oint_{\Sigma} \bar{A}\cdot\bar{n}\,dS = \iiint_{\mathbb{R}^3} \bar{\nabla}\cdot\bar{A}\,dV}$$

$$\rightarrow \iiint_{\mathbb{R}^3} dV \left[ c\rho\frac{\partial U}{\partial t} + \bar{\nabla}\cdot\bar{\phi} - Q \right] = \phi$$

$$+ \qquad + \qquad -$$

$$\boxed{c\rho\frac{\partial U}{\partial t} + \bar{\nabla}\bar{\phi} - Q = 0}$$

$$\bar{\phi} = -K_0\bar{\nabla}U \rightarrow 3\,\text{DIM}$$

$$\phi = -k_0\frac{\partial U}{\partial x} \rightarrow 1\,\text{DIM}$$

× la Temperatura
è Fourier

× la Diffusione
è Fick

Verso Fourier in 3D

$$\boxed{c\rho\frac{\partial U}{\partial t} = \underset{\text{DIV}}{\nabla}\left[k_0\underset{\text{GRAD}}{\bar{\nabla}}U\right] + Q}$$

hp] $K_0 \sim$ cost → $\boxed{\frac{\partial U}{\partial t} = K \nabla^2 U}$, $K = \frac{K_0}{c\rho}$

Fourier in 3D

Laplaciano in coord. euclidee

$$\nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$$

STEADY STATE

$\nabla^2 U = -Q/K_0$ → eq. di Poisson

$\nabla^2 U = 0$ → eq. di Laplace

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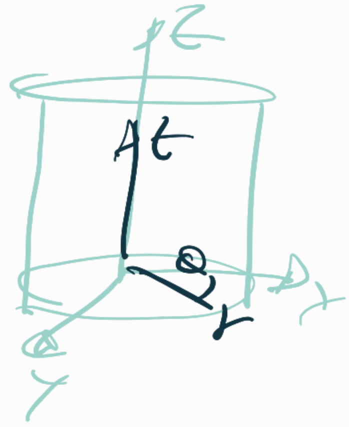
Laplaciano in coordinate cilindriche

$r, \theta, z$

euclideo ← cilindrico

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$




$$\boxed{\nabla^2 U = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2} + \frac{\partial^2 U}{\partial z^2}}$$

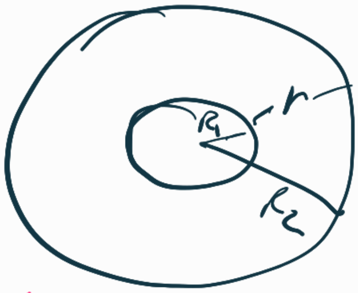
Laplaciano in coordinate sferiche

$$x = \rho \sin\varphi \cos\theta$$
$$y = \rho \sin\varphi \sin\theta$$
$$z = \rho \cos\varphi$$

$$\nabla^2 U = \frac{1}{\rho^2}\frac{\partial}{\partial \rho}\left(\rho^2 \frac{\partial U}{\partial \rho}\right) + \frac{1}{\rho^2 \sin\varphi}\frac{\partial}{\partial \varphi}\left(\sin\varphi \frac{\partial U}{\partial \varphi}\right) + \frac{1}{\rho^2 \sin^2\varphi}\frac{\partial^2 U}{\partial \theta^2}$$

ESERCIZIO 159 Steady STATE

$T(R_2) \equiv T_2$, $T(R_1) \equiv T_1$ Coor. cilindrica




$$\frac{d^2 U}{dx^2} = 0 \Rightarrow \nabla^2 U = 0$$

$$\nabla^2 U = 0 = \cancel{\frac{1}{r}}\partial_r\left[r\frac{\partial U}{\partial r}\right] + \cancel{\frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2}} + \cancel{\frac{\partial^2 U}{\partial z^2}} = 0$$

$$\partial_r\left[r\frac{\partial U}{\partial r}\right] = 0 \Rightarrow r\frac{\partial U}{\partial r} = C_1 \Rightarrow dU = \frac{C_1}{r}dr$$

$U(r) = C_1 \ln r + C_2$ $\qquad U(R_1) = T_1 \quad U(R_2) = T_2$

$$U(r = R_1) = T_1 = C_1 \ln R_1 + C_2 \Rightarrow C_2 = T_1 - C_1 \ln R_1$$

$$U(r = R_2) = T_2 = C_1 \ln R_2 + T_1 - C_1 \ln R_1$$

$$(T_2 - T_1) = C_1 \ln(R_2/R_1) \Rightarrow C_1 = \frac{(T_2 - T_1)}{\ln[R_2/R_1]}$$

$$\Rightarrow U(r) = \frac{T_2 - T_1}{\ln[R_2/R_1]}\ln r + T_1 - \frac{(T_2 - T_1)}{\ln[R_2/R_1]}\ln R_1$$

$$\Rightarrow \boxed{U(r) = \frac{T_2 - T_1}{\ln[R_2/R_1]}\left[\ln\left[\frac{r}{R_1}\right] - 1\right]}$$