Soft glasses, rheology, and trap models

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Ask me questions before dinner
Though I’d rather stay here...
What’s glassy about foam?

Peter Sollich (King’s College London) 

Soft glassy rheology & trap models
What’s glassy about foam?

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Complex fluids/soft matter

- **Foams** (shaving foam, mousse au chocolat)
- **Emulsions** (mayonnaise)
- Dense colloidal suspensions (yogurt, paint)
- Most soft things you can eat (cream cheese, ketchup)
- Clays, pastes, surfactant phases ("onions")

Other important examples (not discussed here):
- Polymers (except star polymers \(\approx\) soft colloids)
- Gels
Science of flow and mechanical deformation

Everything flows (Heraclitus παντα ρει, Dali see above)

Important for e.g. industrial processing

Usefulness of materials ("mouth-feel" for foods, spreading of paint/printing ink)
Complex fluids vs simple fluids

- **Simple fluids** (water): only one lengthscale (atomic/molecular), beyond this continuum theory (Navier-Stokes)

- **Complex fluids**: Hierarchy of scales, e.g. in foam = air bubbles surrounded by fluid films stabilized by soap:
  - Water/soap molecules
  - Film thickness, size of channels where films meet
  - Bubble diameter

- Often intermediate behaviour between **fluid** and **solid**: Shaving foam flows out of a spray can, but doesn’t drip off face
Soft glasses

- Emulsions, dense colloidal suspensions, foams, microgels
- **Structural similarities**: made up of squishy “particles”
- Oil droplets [ignore coalescence], colloidal particles, air bubbles [ignore coarsening]
- Typical particle scale μm, larger for foams, smaller for colloids
- Particles have different shapes and sizes (polydisperse)
- Particle packing is amorphous (disordered)
- Metastable: $k_B T$ too small to make system ergodic & reach optimal packing (crystalline, if polydispersity not too strong)
- So glassy (repulsive glass) – but soft, can easily be made to flow
Outline

1. Rheology: A reminder
2. Soft glasses: Phenomenology and SGR model
3. Intermezzo: Trap models
4. SGR predictions and model limitations
5. Comparison with simulations: Virtual strain analysis
6. Effective temperature dynamics, shear banding
7. Outlook
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Shear stress and strain

- Shear strain: $\gamma = \frac{d}{h}$, shear stress $\sigma = \frac{F}{A}$ (really tensors)
- Elastic solid: $\sigma = G\gamma$, elastic (shear) modulus $G$
- Newtonian fluid: $\sigma = \eta \dot{\gamma}$, viscosity $\eta$
Linear rheology & viscoelasticity

- Small strain increment (step) $\Delta \gamma$ at $t = 0$ causes stress $\sigma(t) = G(t) \Delta \gamma$
- $G(t) =$ stress relaxation function
  - Constant for solid, spike $\eta \delta(t)$ for fluid
- Most materials are in between: viscoelastic
- For short $t$, $G(t)$ nearly constant (solid), but eventually $\rightarrow 0$ (fluid)
- Linear superposition of many small strain steps $\Delta \gamma = \dot{\gamma} \Delta t$:
  $$\sigma(t) = \int_0^t G(t - t') \dot{\gamma}(t') \, dt'$$
Maxwell model

- Elastic solid and viscous fluid “in series” (spring & damper)
- Common stress $\sigma$, elastic strain obeys $\sigma = G_0 \gamma_{el}$, viscous strain $\sigma = \eta \dot{\gamma}_{visc}$
- Total strain rate $\dot{\gamma} = \dot{\gamma}_{el} + \dot{\gamma}_{visc} = \dot{\sigma}/G_0 + \sigma/\eta$
- Solve for small strain step ($\dot{\gamma}(t) = \Delta \gamma \delta(t)$):
  
  $$G(t) = G_0 \exp(-t/\tau), \quad \tau = \eta/G_0$$

- Note $\eta = \int_0^\infty G(t) \, dt$, generally true if(!) flow with constant strain rate is a linear perturbation
Another Maxwell model

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Soft glassy rheology & trap models
Complex modulus

- Experimentally, oscillatory measurements often easier
- If $\gamma(t) = \gamma_0 \cos(\omega t) = \gamma_0 \text{Re } e^{i\omega t}$, then
  \[
  \sigma(t) = \text{Re} \int_{0}^{t} G(t - t')i\omega \gamma_0 e^{i\omega t'} dt' = \text{Re } G^*(\omega) \gamma(t)
  \]

$$G^*(\omega) = i\omega \int_{0}^{\infty} G(t'') e^{-i\omega t''} dt''$$ for large $t$

- Write complex modulus $G^*(\omega) = G'(\omega) + iG''(\omega)$, then
  \[
  \sigma(t) = G'(\omega) \gamma_0 \cos(\omega t) - G''(\omega) \gamma_0 \sin(\omega t)
  \]

- Elastic modulus $G'(\omega)$: in-phase part of stress
- Viscous or loss modulus $G''(\omega)$: out-of-phase (ahead by $\pi/2$)
Complex modulus of Maxwell model

\[ G^* (\omega) = i\omega \times \text{Fourier transform of } G_0 \exp(-t/\tau) = G_0 \frac{i\omega \tau}{1 + i\omega \tau} \]

\[ G' (\omega) = G_0 \frac{\omega^2 \tau^2}{1 + \omega^2 \tau^2}, \quad G'' (\omega) = G_0 \frac{\omega \tau}{1 + \omega^2 \tau^2} \]

- Single relaxation time gives peak in \( G'' (\omega) \) at \( \omega = 1/\tau \)
Creep

- Similar setup can be used when imposing stress and measuring strain
- Step stress: creep compliance $J(t)$

$$\gamma(t) = \int_0^t J(t - t') \dot{\sigma}(t') \, dt'$$

- Oscillatory stress: $\gamma(t) = \text{Re} J^*(\omega) \sigma(t)$
- Consistency with oscillatory strain requires $G^*(\omega) J^*(\omega) = 1$
- Maxwell model: $J^*(\omega) = (1 + i\omega \tau)/G_0$, $J(t) = 1/G_0 + t/\eta$
Nonlinear rheology

- For most complex fluids, steady flow (rate $\dot{\gamma}$) is not a small perturbation, don’t get $\sigma = \eta \dot{\gamma}$
- Flow curve $\sigma(\dot{\gamma})$: stress in steady state
- Often shear-thinning: downward curvature
- Many other nonlinear perturbations:
  - large step stress or strain
  - large amplitude oscillatory stress or strain
  - startup/cessation of steady shear etc
- Most general description: constitutive equation

$$\sigma(t) = \text{some function(al) of strain history } [\gamma(t'), t' = 0 \ldots t]$$
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FIG. 2. The frequency dependence of the storage $G'$ (solid points) and loss $G''$ (open points) moduli of a monodisperse emulsion with $r \approx 0.53 \ \mu m$ for $\phi_{eff} = 0.80$ (diamonds), 0.63 (triangles), and 0.60 (circles). The results for the two larger

- Complex modulus for dense emulsions (Mason Bibette Weitz 1995)
- Almost flat $G''(\omega)$: broad relaxation time spectrum, glassy
Colloidal hard sphere glasses
Mason Weitz 1995

\[ G'(\omega) \text{ (dynes/cm}^2) \]

\[ G''(\omega) \text{ (dynes/cm}^2) \]

\( \phi \)

\( 0.56 \)

\( 0.55 \)

\( 0.53 \)

\( 0.52 \)

\( 0.51 \)

\( 0.50 \)

(a)

(b)

Colloidal hard sphere glasses
Mason Weitz 1995

\[ G'(\omega) \text{ (dynes/cm}^2) \]

\[ G''(\omega) \text{ (dynes/cm}^2) \]

\( \phi \)

\( 0.56 \)

\( 0.55 \)

\( 0.53 \)

\( 0.52 \)

\( 0.51 \)

\( 0.50 \)

(a)

(b)
Figure 3. Schematic representation of an onion phase. $\xi$ is the characteristic length of monodomains. Each monodomain is

- Vesicles formed out of lamellar surfactant phase
- Again nearly flat moduli
Microgel particles
Purnomo van den Ende Vanapalli Mugele 2008

FIG. 1 (color online). $G'$ (open symbols) and $G''$ (solid symbols) of a 7% w/w suspension at 25 °C plotted versus $\omega$ (a) or $\omega t$ (b) for $t_w = 3$ (○), 30 (□), 300 (△), and 3000 s (△). Lines represent the SGR model ($\chi = 0.55$, $G_p = 410$ Pa).

- $G''(\omega)$ flat but with upturn at low frequencies
- **Aging**: Results depend on time elapsed since preparation, typical of glasses
Nonlinear rheology: Flow curves

- Flow curves typically well fitted by \( \sigma(\dot{\gamma}) - \sigma_y \sim \dot{\gamma}^p \) (0 < \( p < 1 \))
- Herschel-Bulkley if yield stress \( \sigma_y \neq 0 \), unsheared state = “glass”
- Otherwise power law flow curve, unsheared state = “fluid” (but \( \eta = \sigma/\dot{\gamma} \rightarrow \infty \) for \( \dot{\gamma} \rightarrow 0 \))
- Shear thinning: \( \sigma/\dot{\gamma} \) decreases with \( \dot{\gamma} \)
A non-glassy model for foam rheology
Princen 1968

- Ideal 2d foam (identical hexagonal cells), $T = 0$
- Apply shear: initially perfectly reversible response, stress increases
- Eventually interfaces rearrange, bubbles “slide”: global yield
- Process repeats under steady shear
- **We get**: yield stress
- **We don’t get**: broad relaxation time spectrum (Buzza Lu Cates 1995), aging
How do we incorporate structural disorder?

Divide sample conceptually into mesoscopic elements

Each has local shear strain $l$, which increments with macroscopic shear $\gamma$

Assumes strain rate $\dot{\gamma}$ uniform throughout system, but allows for variation in local strain and stress (see Barrat & Falk talks)

When strain energy $\frac{1}{2}kl^2$ reaches yield energy $E$, element can yield and so reset to $l = 0$

$k =$ local shear modulus

If all elements have same $E$ and $k$, this would essentially give back the Princen model
New ingredient 1: disorder ⇒ every element has its own $E$

Initial distribution of $E$ across elements depends on preparation

When an element yields, it rearranges into new local equilibrium structure ⇒ acquires new $E$ from some distribution $\rho(E) \propto e^{-E/\bar{E}}$ (assume no memory of previous $E$)

New ingredient 2: Yielding is activated by an effective temperature $x$, to model interactions between elements

$x$ should be of order $\bar{E}$, $\gg k_BT$ (negligible)

Model implicitly assumes low frequency/slow shear: yields are assumed instantaneous, no solvent dissipation
Sketch

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Soft glassy rheology & trap models
Dynamical equation for SGR

- $P(E, l, t)$: probability of an element having yield energy $E$ and local strain $l$ at time $t$
- Master equation ($\Gamma_0 = \text{attempt rate for yields}$)

$$
\dot{P}(E, l, t) = -\gamma \frac{\partial P}{\partial l} - \Gamma_0 e^{-(E-kl^2/2)/x} P + \Gamma(t) \rho(E) \delta(l)
$$

where $\Gamma(t) = \Gamma_0 \langle e^{-(E-kl^2/2)/x} \rangle = \text{average yielding rate}$

- Macroscopic stress $\sigma(t) = k \langle l \rangle$
- Given initial condition $P(E, l, 0)$ and strain history (input) can in principle calculate stress (output)
- We’ll rescale $E, t, l$ so that $\bar{E} = \Gamma_0 = k = 1$; this means also typical yield strains are 1
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Trap model
Bouchaud 1992, also Dean, Monthus ... 

- Without shear ($\dot{\gamma} = 0$), can ignore local strains (all $l \to 0$ eventually)
- Master equation for $P(E,t)$

$$\dot{P}(E,t) = -e^{-E/x} P + \Gamma(t) \rho(E)$$

where $\Gamma(t) = \langle e^{-E/x} \rangle$

- Physical (re-)interpretation: “particle hopping” by activation, in landscape of traps of depth $E$
- Landscape has golfcourse shape: all traps hang off same energy level
- No geometry: every trap connected to every other
Equilibrium & glass transition in the trap model

- Master equation for $P(E,t)$

$$\dot{P}(E,t) = -e^{-E/x}P + \Gamma(t)\rho(E)$$

- $P(E,t)$ approaches equilibrium $P_{eq}(E) \propto \exp(E/x)\rho(E)$ for long $t$ (Boltzmann distribution; $E$ is measured downwards)

- Get glass transition if $\rho(E)$ has exponential tail
  (possible justification from extreme value statistics)

- Reason: for low enough $x$, $P_{eq}(E)$ cannot be normalized

- For $\rho(E) = e^{-E}$ this transition happens at $x = 1$

- For $x < 1$, system is in glass phase; never equilibrates

- Aging: evolution into ever deeper traps
Aging in the trap model

- Easier in terms of lifetimes $\tau = \exp(E/x)$
- Then $\rho(\tau) \sim \tau^{-x-1}$, $P_{eq}(\tau) \sim \rho(\tau)\tau \sim \tau^{-x}$
- Assume initial condition $P(\tau, 0) = \rho(\tau)$
- At age $t_w$, particle hasn’t hopped if initial $\tau \gg t_w$; traps with $\tau \ll t_w$ have become equilibrated:

$$P(\tau, t_w) \propto \begin{cases} \tau^{-x} & \text{for } \tau \ll t_w \\ t_w \tau^{-x-1} & \text{for } \tau \gg t_w \end{cases}$$

- Normalization: for $x > 1$, most “mass” for $\tau = O(1)$, $P(\tau, t_w) \to \text{const} \times \tau^{-x}$ for large $t_w$
- For $x < 1$, in glass phase, most mass for $\tau = O(t_w)$
- Then get scaling form $P(\tau, t_w) = (1/t_w)f(\tau/t_w)$: typical relaxation times $\sim t_w$, simple aging
Aging in the trap model: Sketch

\[ P(\tau, t, t) \]

- (a) \( x = 1.3 \)
- (b) \( x = 0.7 \)

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Trap model: Interpretations and uses

- Originally proposed by Bouchaud as model for motion in phase space (spin glasses)
- Simple(st?) aging mechanism, controlled by energy barriers and activation
- Distinct from mean-field spin glasses, aging controlled by entropy barriers (rare downhill directions), $T$ not crucial
- Connection with real-space dynamics?
  Small subsystems $\approx$ independent trap models (Heuer et al)
- Intriguing fluctuation-dissipation behaviour (see my webpage)
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Linear response in the fluid phase

- Calculation yields average of Maxwell models:
  \[ G^*(\omega) = \left\langle \frac{i\omega \tau}{1 + i\omega \tau} \right\rangle, \text{ average is over } P_{eq}(\tau) \]
- For large \( x \), get usual power-law dependences for small \( \omega \)
- But near \( x = 1 \) get \( G' \sim G'' \sim \omega^{x-1} \): both become flat
Linear response: Aging
Sollich PS Cates 2000

- Conceptual issue: with aging, $G^*(\omega) \rightarrow G^*(\omega, t, t_w)$
- $G^*(\omega, t, t_w)$ could depend on final time $t$
  and start time $t_w$ of shear
- Luckily, dependence on $t_w$ is weak: $G^*(\omega, t)$
- “Inherits” simple aging $1/\omega \sim t$: $G^*(\omega, t) \sim 1 - (i\omega t)^{x-1}$
Linear response: Aging

\[ G(\omega) \]

\[ \omega \]

- \( x = 1.3 \)
- \( x = 1.0 \)
- \( x = 1.0 \)
- \( x = 0.7 \)
Comparison with experiments on microgel particles
Purnomo van den Ende Vanapallli Mugele 2008

(a) $G', G''$ (Pa)

T=35 °C
x=0.67±0.03

(b) $G', G''$ (Pa)

T=37 °C
x=0.87±0.03

(c) $G', G''$ (Pa)

T=38 °C
x=2.2±0.3

(d) $G', G''$ (Pa)

T=40 °C
x>3
Calculation: steady state, so set $\dot{P} = 0$ in master equation, integrate differential eqn for $l$; $\Gamma$ from normalization (try it)

Three regimes for small $\dot{\gamma}$:

$$\sigma \sim \begin{cases} 
\dot{\gamma} & \text{for } 2 < x : \quad \text{Newtonian} \\
\dot{\gamma}^{x-1} & \text{for } 1 < x < 2 : \quad \text{power law} \\
\sigma_y(x) + \dot{\gamma}^{1-x} & \text{for } x < 1 : \quad \text{Herschel-Bulkley}
\end{cases}$$
Relation between flow and aging

- No aging in steady flow
- Driving by shear restores ergodicity
- Flow interrupts aging (Kurchan)
Yield stress

- Yield stress increases **continuously** at glass transition
- Compare MCT prediction: **discontinuous** onset of yield stress
- Physics?
  - Elastic networks/stress chains vs caging?
  - Jamming transition vs glass transition?
- Could e.g. emulsions exhibit both transitions?
General nonlinear rheology: Constitutive equation

PS 1998

- SGR model can be solved: switch variable $l \rightarrow l - \gamma(t)$ to eliminate $\partial P/\partial l$ term, then integrate (try it)

- For simplest initial condition $P(E, l, 0) = \rho(E)\delta(l)$ get constitutive equation (2nd equation determines $\Gamma(t)$)

\[ \sigma(t) = \gamma(t)G_\rho(Z(t, 0)) + \int_0^t \Gamma(t')[\gamma(t) - \gamma(t')]G_\rho(Z(t, t')) \, dt' \]

\[ 1 = G_\rho(Z(t, 0)) + \int_0^t \Gamma(t')G_\rho(Z(t, t')) \, dt' \]

- $G_\rho(t) = \int \rho(E) \exp(-te^{-E/x}) \, dE$ survival probability

- $Z(t, t') = \int_{t'}^t \exp([\gamma(t'') - \gamma(t')]^2/2x)$ effective time,

  $Z(t, t') = t - t'$ for small strains

- Overall interpretation as birth-death process
Example: Large amplitude oscillatory strain

- Close to but above glass transition \( (x = 1.1, \omega = 0.01) \)
- Increasing strain amplitude gives stronger **nonlinearities**
- Hysteresis-like loops
Large oscillatory strain: Complex modulus

- $G''$ first increases with amplitude, becomes larger than $G'$
- Large strain **fluidizes** an initially predominantly elastic system
- Compare experiments on colloidal hard spheres (right)
- Quantitative comparison for foam
  (Rouyer Cohen-Addad Höhler PS Fielding 2008)
SGR predictions: Summary

- **Flow curves**: Find both Herschel-Bulkley ($x < 1$) and power-law ($1 < x < 2$)
- **Viscoelastic spectra** $G'$, $G'' \sim \omega^{x-1}$ are flat near $x = 1$
- In glass phase ($x < 1$) find rheological aging, loss modulus $G'' \sim (\omega t)^{x-1}$ decreases with age $t$
- **Steady shear** always interrupts aging, restores stationary state
- Large amplitude $G'$ and $G''$ show fluidization behaviour similar to experiments
- Stress overshoots in shear startup, linear and nonlinear creep, rejuvenation and overaging (Lequeux, Viasnoff, McKenna, Cloître, Roettler . . . )
Limitations of SGR model

- **Scalar model** with ideal local elasticity up to yield – both can be fixed (Cates PS 2004)
- No spatial information: geometry of stress redistribution might be important, also non-affine flow (Barrat talk)
- **Length scale** of elements: needs to be large enough to allow local strain and stress to be defined, but otherwise unspecified
- Interpretation of effective temperature $x$? Link to material parameters? Should have own dynamics? (see later)
- What sets fundamental **time scale** (attempt rate for yielding)?
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Simulations to the rescue?

- Can we use simulation data to:
  - See how far the SGR model represents physical reality?
  - Get better understanding of model parameters?
  - Tell us where we should improve the model?
  - Need to develop method for explicit coarse-graining of simulation data
Defining elements

- Focus on $d = 2$ ($d = 3$ can be done but more complicated)
- Make elements circular to minimize boundary effects
- Position circle centres on square lattice to cover all of the sample (with some overlap)
- Once defined, element is co-moving with strain: always contains same particles
- Avoids sudden change of element properties when particles leave/enter, but makes sense only up to moderate $\Delta \gamma$
- Measuring average stress in an element is easy but how do we assign strain $l$, yield energy etc for a given snapshot?
Virtual strain analysis

- Deliberately want local yield points etc: interaction between elements is accounted for separately within model
- Cannot “cut” an element out of sample and then strain until yield – unrealistic boundary condition
- Idea: Use rest of sample as a frame
- Deform the frame affinely to impose a virtual strain $\tilde{\gamma}$
- Particles inside element relax non-affinely to minimize energy
- Gives energy landscape $\epsilon(\tilde{\gamma})$ of element
- Yield points are determined (for $\tilde{\gamma} > 0$ and $< 0$) by checking for reversibility for each small $\Delta\tilde{\gamma}$ (adaptive steps)
Example: Virtual strain sequence 1
Example: Virtual strain sequence 2
Example: Virtual strain sequence 3
Example: Virtual strain sequence 4
Example: Virtual strain sequence 5
Example: Virtual strain sequence 6
Example: Virtual strain sequence 7
Example: Virtual strain sequence 8
Example: Virtual strain sequence 9
Element energy landscape

Extract: minimum energy $\epsilon_{\text{min}}$, strain away from local minimum $l = -\tilde{\gamma}_{\text{min}}$, yield strains $\gamma_{\pm}$, yield barriers $E_{\pm}$
Local modulus

Quadratic fit of energy near minimum, or linear fit of stress, gives local modulus $k$.
Systems studied

- **Polydisperse Lennard-Jones mixtures** (Tanguy et al), quenched to low temperatures \( T = 0.005 \ll T_g \)
- Low shear rates \( \dot{\gamma} \sim 10^{-3} \); \( N = 10^4 \) particles at \( \rho = 0.95 \)
- Steady shear driven from the walls (created by “freezing” particles in top/bottom 5% some time after quench)
- Check for stationarity & affine shape of velocity profile before taking data
- Each element contains \( \approx 40 \) particles (diameter = 7): large enough to have near-parabolic energy landscape, small enough to avoid multiple local yield events inside one element
Simulation demo
Close-up
Exponential tail; detailed form can be fitted by SGR model
Yield strain distributions

Symmetric as assumed in SGR; gap around 0 or maybe power-law approach (exponent $\approx 4$)
Clear spread; not constant as assumed in model.
But yield strains $\gamma_\pm$ still controlled by $E_\pm$; no correlation with $k$.  

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Soft glassy rheology & trap models
Negative $l$, need to extend SGR to allow frustration: $l \neq 0$ after yield ($\delta(l) \to \rho(l|E) \propto (1 - kl^2/2E)^b$)
Dynamics: Evolution of local strain with time

Typical sawtooth shape assumed by SGR
Population picture of $l$-dynamics

Scatter plot of $l$(after $\Delta \gamma$) vs $l$(initial)

Separation into strain convection and yield events

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Same for larger $\Delta \gamma$
...and yet larger

\[ \Delta \gamma = 0.020 \]
Change in other landscape properties
Example of modulus

Stays largely constant between yields as expected; same for yield barriers etc

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Comparing real and virtual deformations

Primary yield

Curve: virtual energy landscape.
Vertical lines: Real $\epsilon$ versus $l - l_0$ for uniform steps $\Delta \gamma$
Good match, even for energy drop after yield
Comparing real and virtual deformations (cont)

Induced yield

Curve: virtual energy landscape.
Vertical lines: Real $\epsilon$ versus $l - l_0$ for uniform steps $\Delta \gamma$
Summary for virtual strain analysis

- **Virtual strain method** for assigning local strains, yield energies
- **Generic**: can be used on configurations produced by any (low-$T$) simulation
- Steady state distributions in shear flow seem in line with SGR (detailed fits in progress), though e.g. local modulus $\neq$ const
- Dynamics of local strain has typical sawtooth shape; local strain rate is of same order as global one but not identical
- Energy landscapes for real and virtual deformations match (but not purely quadratic)
- To do: analysis of induced yield events – well modelled by effective temperature?
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Effective temperature dynamics
Fielding Cates PS 2008

- Shouldn’t effective temperature \( x \) be determined self-consistently by dynamics?
- To allow for potential shear banding, split sample in \( y \) (shear gradient)-direction
- Separate SGR model for each \( y \), with \( x(y) \)
- Relaxation-diffusion dynamics:

\[
\tau_x \dot{x}(y) = -x(y) + x_0 + S(y) + \lambda^2 \frac{\partial^2 x}{\partial y^2}
\]

- \( x \) is “driven” by energy dissipation rate:

\[
S = a \langle l^2 \exp\left(-\left[\frac{(E - l^2/2)}{x}\right]\right) \rangle
\]

- Assume that \( x \) equilibrates (locally) quickly: \( \tau_x \to 0 \)
Flow curve

\[ a = 2, \ x_0 = 0.3 \]

- Steady state: \( x = x_0 + 2a\sigma(x, \dot{\gamma})\dot{\gamma} \)
- Shear startup with imposed mean \( \dot{\gamma} \) across sample: shear banding
Nature of banded state

\[ a = 2, \quad x_0 = 0.3, \quad \dot{\gamma} = 0.05, 0.1, 0.2 \]

- "Hot" band: \( \dot{\gamma} > 0 \), ergodic
- "Cold" band: \( \dot{\gamma} = 0 \), aging
Viscosity bifurcation at imposed stress
Coussot, Bonn, ...
\( x \) now driven by yield rate, \( S \propto \langle \exp(-[(E - l^2/2)/x]) \rangle \)

- **Hysteresis** in shear rate sweep: banding on way up, stay on fluid branch on way down
- Resembles data for multi-arm polymers (Holmes Callaghan Vlassopoulos Roovers 2004)
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Summary & Outlook

- **Trap models** for aging dynamics in glasses, focus on activation
- **SGR model** adds strain to this & re-interprets trap depths as yield energies
- Reproduces much (not all) of *rheological behaviour* of soft glasses
- ...and some cytoskeletal rheology(?)
- **Virtual strain** method allows detailed comparison with simulations: some encouraging agreement, but also suggests modifications
- **Dynamics of \( x \)**: phenomenological models useful, but too much choice?
- **To do**: linking to other approaches (STZ, Picard et al); coarse-graining from “microscopic” models?