Chapter 1

Motifs stability in hierarchical modular networks

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Recent advances in our understanding of information processing in biological systems have highlighted the importance of modularity in the underlying networks (ranging from metabolic to neural networks), as well as the crucial existence of *motifs*, namely small circuits (not necessary loopy) whose empirical presence in these networks is statistically high. In these notes, mixing statistical mechanical with graph theoretical perspectives and restricting on hierarchical modular networks, we analyze the stability of key motifs that naturally emerge and we prove that all the loopy structures have systematically a broader steadiness with respect to loop-free motifs.

Introduction

Network theory, coupled with Statistical Mechanics, is becoming a crucial tool for investigating Biological Complexity and, following this approach, crucial questions, ranging from intra-cellular investigations (as for instance in metabolic or protein networks[1,12,16,17]), to extra-cellular ones (as for instance in neural networks[4,9,20,22]) have already been satisfactorily addressed.

According to empirical evidence, biological networks typically exhibit scale-free and/or hierarchical topologies[18,23,24] with a high number (with respect to a random reference) of "motifs", namely recurrent and stronglyconnected sub-graphs or patterns.[19] Further, the interaction strength (e.g., based on lock-and-key mechanisms[4]) between the elements making up the network usually varies over several orders of magnitude, in accordance with a log-normal or power-law distribution for link's magnitudes. This has stimulated a renewed interest for the Dyson model:[14] indeed the latter, originally developed as a model to overcome mean-field limitations in the statistical mechanical description of ferromagnetism, is exactly a hierarchical network, where spins are pasted on its nodes and their couplings follow a power-law distribution.[8]

In these notes we aim to analyze the meta-stability of key motifs highly occurring in the Dyson model, as their existence has been found only recently.[2,3] In particular, we consider the *dimer*, i.e., the prototype of a loopless reticular animal, and the *square*, i.e., the prototype of a loopy reticular animal, and we check whether magnetic configurations where spins associated to these motifs are misaligned with respect to the bulk -but aligned among themselves- are stable. Not surprisingly, while the former is found to be always unstable (i.e. there is no value of the tuneable parameters defining the model that allows its stability), the latter has a range of stability. It is worth noting, however, that -as these motifs are by definition not-extensive (i.e. their sizes do not scale with the system size)- nor they contribute to the model's free energy in the thermodynamic limit, neither they are expected to be stable whenever a finite-amount of fast noise is applied on the system.

As a last remark, we note that the Dyson model has a power law distribution for the link's magnitude[8] as well as a modular architecture of the graph hosting the spins:[3] remarkably, the reason for the stability of its loopy motifs lies exactly in these intrinsic features of the model, that -in turn- play a major role even in real biological networks,[15] exactly those where the presence of motifs is expected.[18,19]

1. Definition of the model

The aim of this section is to give a microscopical description of the Dyson Hierarchical Model (DHM), which is composed by 2^{k+1} Ising spins S_i, for $i = 1, \cdots, 2^{k+1}$, that are embedded in a hierarchical topology. The model is represented by the Hamiltonian introduced in the following definition:

Definition 1. *The Hamiltonian of Dyson's Hierarchical Model (DHM) is*

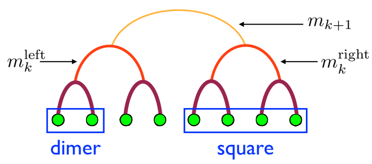

Fig. 1. Schematic representation of the hierarchical network defined by the Dyson model of size 4. Each node hosts an Ising spin and interactions among nodes are stronger (here denoted by thicker links) for closer nodes as depicted in the Hamiltonian (eq.(1)) defining the model.

defined by

$$H_{k+1}(\vec{S}|J,\sigma) = H_k(\vec{S_1}) + H_k(\vec{S_2}) - \frac{J}{2^{2\sigma(k+1)}} \sum_{i<j=1}^{2^{k+1}} S_i S_j, \tag{1}$$

where $J > 0$ *and* $\sigma \in (1/2, 1)$ *are numbers tuning the interaction strength. Clearly* $\vec{S_1} \equiv \{S_i\}_{1 \le i \le 2^k}$, $\vec{S_2} \equiv \{S_j\}_{2^k+1 \le j \le 2^{k+1}}$ *and* $H_0[S] = 0$.

Thus, in this model, σ triggers the decay of the interaction with the distance among spins, while J uniformly rules the overall intensity of the couplings. Note further that the coupling distribution $P(J)$ is scale free as it follows the power-law relation:[8] $P(J) \propto J^{-\frac{1}{2\sigma}}$. We can introduce the partition function $Z_{k+1}(\beta, J, \sigma)$ at finite volume $k+1$ as

$$Z_{k+1}(\beta, J, \sigma) = \sum_{\sigma}^{2^{k+1}} \exp\left[-\beta H_{k+1}(\vec{S}|J,\sigma)\right], \tag{2}$$

and the related free energy $f_{k+1}(\beta, J, \sigma)$, namely the intensive logarithm of the partition function, as

$$f_{k+1}(\beta, J, \sigma) = \frac{1}{2^{k+1}} \log \sum_{\vec{S}} \exp\left[-\beta H_{k+1}(\vec{S}) + h \sum_{i=1}^{2^{k+1}} S_i\right]. \tag{3}$$

We introduce also the global magnetization $m = \lim_{k\to\infty} m_{k+1}$ where

$$m_{k+1} = \frac{1}{2^{k+1}} \sum_{i=1}^{2^{k+1}} S_i, \tag{4}$$

that can be defined recursively, level by level (see Figure One). Finally, we denote the thermodynamical average as

$$\langle m_{k+1}(\beta, J, \sigma)\rangle = \frac{\sum_{\vec{S}} m_{k+1} e^{-\beta H_{k+1}(\vec{S}|J,\sigma)}}{Z_{k+1}(\beta, J, \sigma)}, \tag{5}$$

and $\lim_{k\to\infty}\langle m_{k+1}(\beta, J, \sigma)\rangle = \langle m(\beta, J, \sigma)\rangle$.

We are interested in understanding the conditions to be applied on σ such that different configurations remain stable in noiseless conditions. We will start with some simple cases, and we will try to apply the results to a general structure composed by 2^n elements, with $n < k+1$.

2. Loop-less case: Stability analysis of the dimer

The goal of this section is to study the existence of possible values of σ such that the dimer (i.e. a spin-configuration where $S_i = +1$ for $i = 1, 2$ and $S_j = -1$ for $j = 3, \cdots, k+1$) remains stable, clearly in the noiseless limit. To reach our conclusions, we adapt to the case an interpolative strategy -firstly developed in[13]- that has been recently applied to hierarchical networks:[2]

Definition 2. *Once considered a real scalar parameter $t \in [0, 1]$, we introduce the following interpolating Hamiltonian*

$$H_{k+1,t}(\vec{S}) = -\frac{Jt}{2^{2\sigma(k+1)}} \sum_{i>j=1}^{2^{k+1}} S_i S_j - \frac{Jm(1-t)}{2^{(2\sigma-1)(k+1)}} \sum_{i=1}^{2^{k+1}} S_i + +H_k(\vec{S_1}) + H_k(\vec{S_2}), \tag{6}$$

such that for $t = 1$ the original system is recovered, while at $t = 0$ the two-body interaction is replaced by an effective, tractable one-body term. The possible presence of an external magnetic field can be accounted simply by adding to the Hamiltonian a term $\propto h\sum_i^{2^{k+1}} \sigma_i$, with $h \in \mathbb{R}$.

This prescription allows defining an extended partition function as

$$Z_{k+1,t}(h, \beta, J, \sigma) = \sum_{\vec{S}} \exp\{-\beta[H_{k+1,t}(\vec{S}) + h\sum_{i=1}^{2^{k+1}} S_i]\}, \tag{7}$$

where the subscript t stresses its interpolative nature, and, analogously,

$$\Phi_{k+1,t}(h, \beta, J, \sigma) = \frac{1}{2^{k+1}} \log Z_{k+1,t}(h, \beta, J, \sigma). \tag{8}$$

It is easy to show that

$$\Phi_{k+1,0}(h, \beta, J, \sigma) = \Phi_{k,1}(h + mJ2^{(k+1)(1-2\sigma)}, \beta, J, \sigma), \tag{9}$$

so that we can write

$$\Phi_{k+1,1}(h,\beta,J,\sigma) = \Phi_{k+1,0}(h,\beta,J,\sigma) + \int_0^1 \frac{d\Phi}{dt}dt \Longrightarrow$$
$$\Phi_{k+1,1}(h,\beta,J,\sigma) = \Phi_{k,1}(h + mJ2^{(k+1)(1-2\sigma)},\beta,J,\sigma) + \int_0^1 \frac{d\Phi}{dt}dt. \quad (10)$$

Using the identity (10), with the appropriate computations, we obtain

$$\begin{aligned}\Phi_{k+1,1}(h) &= \Phi_{k+1,0}(h) - \frac{\beta J}{2}(2^{(k+1)(1-2\sigma)}m^2 + 2^{-2(k+1)\sigma}) + \\ &+ \frac{\beta J}{2}2^{(k+1)(1-2\sigma)}\left\langle (m_{k+1}(\vec{S}) - m)^2 \right\rangle_t \geq \\ &\geq \Phi_{k,1}(h + Jm2^{(k+1)(1-2\sigma)}) - \frac{\beta J}{2}(2^{(k+1)(1-2\sigma)}m^2 + \\ &+ 2^{-2(k+1)\sigma}). \end{aligned} \quad (11)$$

We already know that we can study non-standard stabilities where the system undergoes the influence of two different contributions m_1 and m_2, with the same absolute value, but opposite in sign.[2] Now we want to analyze the case in which the subsystems have different cardinality: in this case, the first one is constituted only by two spins, and the other one is constituted by all the others; we can write the two sub-magnetizations m_1 and m_2, more rigorously, as:

$$m_1 = \frac{2}{2^{k+1}}\sum_{i=1}^{2} S_i, \qquad m_2 = \frac{2^{k+1}-2}{2^{k+1}}\sum_{i=3}^{2^{k+1}} S_i, \quad (12)$$

where m_1 has an opposite sign with respect to m_2. This means that the system preserves the same general magnetization m up to the last level, where one has 2^k blocks formed by 2 spins: at this point, one of the blocks, say the one formed by S_1 and S_2, is not affected by the rest of the system, while all the others 2^{k-1} blocks continue to interact each others. Rigorously, starting from the formula (11) and iterating the same interpolating estimates up to the first level (i.e. the level in which we are considering spins at distance $d_{ij} = 1$) we arrive to

$$\Phi_{k+1,1}(h) \geq \Phi_{1,0}(h + Jm\sum_{l=2}^{k+1} 2^{l(1-2\sigma)}) - \frac{\beta J}{2}(\sum_{l=2}^{k+1} 2^{l(1-2\sigma)}m^2 + \sum_{l=2}^{k+1} 2^{-2l\sigma}). \quad (13)$$

Now, asking for dimer stability, we assume that

$$\Phi_{1,0}(h + Jm\sum_{l=2}^{k+1} 2^{l(1-2\sigma)}) = \frac{1}{2^k}\Phi_{1,0}^1(h + Jm\sum_{l=2}^{k+1} 2^{l(1-2\sigma)}) + \\ + \frac{2^{k+1}-2}{2^{k+1}}\Phi_{1,0}^2(h + Jm\sum_{l=2}^{k+1} 2^{l(1-2\sigma)}), \quad (14)$$

where we wrote the two contributions to the interpolating free energy from different groups of spins.

With some straightforward calculation we obtain that

$$\frac{1}{2^k}\Phi_{1,0}^1(h + Jm\sum_{l=2}^{k+1} 2^{l(1-2\sigma)}) = \frac{1}{2^k}\Phi_{0,1}^1(h + m_1 J 2^{1-2\sigma} + mJ\sum_{l=2}^{k+1} 2^{l(2\sigma)}) \\ = \frac{1}{2^k}\log\cosh\Big[\beta J(m_1 2^{1-2\sigma} + h + m\sum_{l=2}^{k+1} 2^{l(1-2\sigma)})\Big] + \\ + \frac{1}{2^k}\log 2, \quad (15)$$

and this allows writing

$$f_{k+1} \geq \log 2 + \frac{1}{2^k}\log\cosh\Big[\beta J(m_1 2^{1-2\sigma} + h + m\sum_{l=2}^{k+1} 2^{l(1-2\sigma)})\Big] + \\ + \frac{2^{k+1}-2}{2^{k+1}}\log\cosh\Big[\beta J(m_2 2^{1-2\sigma} + h + m\sum_{l=2}^{k+1} 2^{l(1-2\sigma)})\Big] + \\ - \frac{\beta J}{2}(\sum_{l=2}^{k+1} 2^{l(1-2\sigma)}m^2 + \sum_{l=2}^{k+1} 2^{-2l\sigma}) + \\ - \frac{\beta J}{2}2^{1-2\sigma}\Big(\frac{2m_1^2 + (2^{k+1}-2)m_2^2}{2^{k+1}}\Big). \quad (16)$$

Since we are looking for conditions on σ for the stability of the dimer, to ensure that the term with m_1 is much more significant than the term with m in the logarithm, we need to fullfill

$$\sum_{l=2}^{k+1} 2^{l(1-2\sigma)} < 2^{1-2\sigma}. \quad (17)$$

In the thermodynamic limit of the system (i.e. when $k \to +\infty$), this means

$$\sum_{l=2}^{\infty} 2^{l(1-2\sigma)} < 2^{1-2\sigma} \Longleftrightarrow \frac{2^{2(1-2\sigma)}}{1-2^{1-2\sigma}} < 2^{1-2\sigma} \Longleftrightarrow 2^{1-2\sigma} < 1 - 2^{1-2\sigma} \Longleftrightarrow$$
$$2^{2-2\sigma} < 1 \Longleftrightarrow 2-2\sigma < 0 \Longleftrightarrow 2\sigma > 2 \Longleftrightarrow \sigma > 1. \tag{18}$$

The condition (18) shows that the dimer can be stable only if $\sigma > 1$, but this bound violates the condition $\sigma \in (1/2,\ 1)$, as stated in the definition of the model. This result can be expressed in the following

Theorem 1. *We can not find conditions on σ such that a misaligned dimer remains stable in the noiseless limit.*

Remark 1. *It is possible to observe that the first term on the right of the (16) goes to zero as k tends to infinity. This means that in the thermodynamic limit the contribute of the dimer becomes negligible in the total free energy as it should.*

3. Loopy case: Stability analysis of the square

Here we focus on the structure made by spins whose configuration is $S_i = +1$ when $i = 1, \cdots, 4$ and $S_j = -1$ when $j = 5, \cdots, 2^{k+1}$, and we will see that now there exist some possible values of σ such that this configuration remains stable.

We recall that the mean-field interpolating Hamiltonian can be written as in (6), and the partition function and the free energy are respectively like in (7) and (8). Proceeding as already done for the dimer, using (9) and (10), we can imagine that all the spins in the system maintain the same magnetization up to the second level, and then we have two different contributions: the first one given by a group composed by only four spins, and the second one given by all the other ones. Then, in this case, we can write

$$m_1 = \frac{2^2}{2^{k+1}} \sum_{i=1}^{4} S_i \qquad m_2 = \Big(\frac{2^{k+1}-4}{2^{k+1}}\Big) \sum_{i=5}^{2^{k+1}} S_i. \tag{19}$$

With the same computations explained for the dimer, considering this new configuration, we arrive to

$$f_{k+1}(h,\beta,J,\sigma) \geq \log 2 + \frac{4}{2^{k+1}} \log\cosh(\beta J(m_1 \sum_{l=1}^{2} 2^{l(1-2\sigma)} + h + m \sum_{l=3}^{k+1} 2^{l(1-2\sigma)})) +$$
$$+ \frac{2^{k+1}-4}{2^{k+1}} \log\cosh(\beta J(m_2 \sum_{l=1}^{2} 2^{l(1-2\sigma)} + h + m \sum_{l=3}^{k+1} 2^{l(1-2\sigma)})) +$$
$$- \frac{\beta J}{2}(\sum_{l=3}^{k+1} 2^{l(1-2\sigma)} m^2 + \sum_{l=3}^{k+1} 2^{-2l\sigma}) - \frac{\beta J}{2} \sum_{l=1}^{2} 2^{l(1-2\sigma)} \Big(\frac{m_1^2 + m_2^2}{2}\Big).$$

Again, investigating the values of σ for the stability of the square, we can state the following theorem

Theorem 2. *A misaligned square remains stable in the thermodynamic limit when* $\sigma \in (\frac{3}{4}, 1]$.

Proof. As shown in the previous sections, We start asking

$$\sum_{l=3}^{k+1} 2^{l(1-2\sigma)} < \sum_{l=1}^{2} 2^{l(1-2\sigma)},$$

to ensure that the contribution given by $m_{1,2}$ is more significant than the one given by m. In the thermodynamic limit the previous inequality turns out to reads as

$$\sum_{l=3}^{+\infty} 2^{l(1-2\sigma)} < \sum_{l=1}^{2} 2^{l(1-2\sigma)}.$$

By computing this sum we have that

$$\frac{2^{3(1-2\sigma)} - 2^{(k+2)(1-2\sigma)}}{1-2^{1-2\sigma}} < 2^{1-2\sigma}(1 + 2^{1-2\sigma}),$$

that is $\sigma > 3/4$.

Since $\sigma \in (\frac{1}{2}; 1]$ by definition, differently from the dimer configuration, we found an interval of values of σ such that the square remains stable.

□

We can conclude that, moving from the dimer to the square, we find crucial differences: while the first configuration can never be stable in the thermodynamic limit, the second one -under appropriate conditions on σ- remains stable. It is worth noticing that the model free energy is obtained summing over all the contributions, thus even those from both the subsystems, however -in the thermodynamic limit- their contributions are vanishing as expected, thus (finite-size) motifs, while crucial for dynamical processes on networks, do not contribute to its thermodynamics.

4. Generalizations to other loopy motifs

In the previous sections we have proved that loopy motifs can be generically stable in hierarchical networks with scale-free distributed couplings, while less structured motifs without loops in general can not. Scope of the present section is to extend the previous results toward more general loopy motifs, showing that the procedure used to find the critical value of σ, can be extended to find the stability of other configurations. In particular, we are going to analyze the general case where

$$m_1 = \frac{2^n}{2^{k+1}} \sum_{i=1}^{2^n} S_i, \qquad m_2 = \frac{2^{k+1} - 2^n}{2^{k+1}} \sum_{i=2^n+1}^{2^{k+1}} S_i, \qquad n \in [1, k+1]. \quad (20)$$

Proceeding exactly as for the square configuration, we imagine that at the n-th level, with $n \in [2, k]$, the two groups, one constituted by 2^n spins, and the other one by $2^{k+1} - 2^n$ have different magnetization. Following step-by-step the general scheme and calculations outlined in the previous sections we can finally state

Theorem 3. *The general configuration of 2^n spins, where they are arranged according to eq. 20, remains stable in the thermodynamic limit when $\sigma \in (\frac{n+1}{2n}, 1]$ with $n \in [1, k]$.*

Remark 2. *The condition $\frac{n+1}{2n} \leq 1$ is verified for all $n \geq 1$.*

Remark 3. *Via the route paved in these notes we can infer the range of validity of σ in a novel way. Indeed imposing that $n = k$, and then taking the limit $k \to \infty$, this results in the condition that $\sigma \in (\frac{1}{2}, 1]$.*

5. Conclusions and outlooks

At a *macroscopic scale*, from a graph theory perspective, several biological networks are scale-free and show modularity,[15,17,20,21] while, at the *microscopical scale*, these networks are characterized by the presence of highly expressed reticular motifs.[19,23]
In these notes, driven by a statistical mechanical guide, we analyzed the

stability of motifs within the Dyson model,[14] as the latter can be read as a hierarchical network where modules naturally emerge and whose distribution of coupling strength is scale-free.[2,8]
Of course, the exhaustion of motif's exploration -already confined within this model- is out of the scope of the present notes as here we restricted the analysis to the prototype of a loop-free structure, that is the *dimer*, and to the prototype of a loopy structure, that is the *square*, while tracing out the general guidelines to be followed to generalize further investigations on Dyson network's motifs at will.
In particular, we successfully proved that, while the loop-free motif always lacks stability and will always fluctuate, for the latter -the loopy motif- there is a region of stability (in the space of the tuneable parameters), in agreement with intuition and with previous results from the Literature.
Next efforts will be devoted to a systematic exploration of network's motifs within the modular-hierarchical as well as within more general scale-free networks.

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