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Toward a quantitative approach to migrants integration

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Abstract – Migration phenomena and all the related issues, like integration of different social groups, are intrinsically complex problems since they strongly depend on several competitive mechanisms as economic factors, cultural differences and many others. By identifying a few essential assumptions, and using the statistical mechanics of complex systems, we propose a novel quantitative approach that provides a minimal theory for those phenomena. We show that the competitive interactions in decision making between a population of N host citizens and P immigrants, a bi-partite spin-glass, give rise to a *social consciousness* inside the host community in the sense of the associative memory of neural networks. The theory leads to a natural quantitative definition of migrant's "integration" inside the community. From the technical point of view this minimal picture assumes, as control parameters, only general notions like the strength of the random interactions, the ratio between the sizes of the two parties and the cultural influence. Few steps forward, toward more refined models, which include a digression on the kind of the felt experiences and some structure on the random interaction topology (as dilution to avoid the plain mean-field approach) and correlations of experiences felt between the two parties (biasing the distribution of the coupling) are discussed at the end, where we show the robustness of our approach.

Introduction. – On November 2008, The European Parliament has approved the introduction of a "blue card" based on the American green-card counterpart. This issue relies heavily on the introduction of qualified workers from third countries, calling even more attention to the subject of "immigration".

Focusing only on EU for simplicity, as USA scenarios are intrinsically more complex and tangled [1], residents of one member nation of the European Union are allowed to work in other member nations with little to no restriction on movement. Due to this policy, traditionally homogenous countries which usually sent a significant portion of their population overseas, such as Italy, are seeing an influx of immigrants from several countries with lower per capita annual earning rates, triggering nationwide immigration debates [2].

Barriers to migration are set not only in legal form; as a matter of fact, social opposition to immigration in Europe is visible, the anti–mass-immigration perspective is predominantly nationalist and cultural, rather than economic (on the contrary the latter is often improved by migration phenomena [3]), conferring to this issue a social label.

Again, as a matter of fact, continuous immigration rates persist and host people is not left unaffected by this: judgements with respect to immigrants are continuously raised as well as debated inside the community; further, beyond a "peer-to-peer" interaction among citizens, media share a certain uniform influence in orienting popular opinion on the topic [4].

Immigration phenomena, as an aspect of complex social behavior, surely should match the common properties of complexity [5,6], and, as in the whole social science [7,8], a lot of attention in their modelling has been achieved. Theories have been derived by the so-called "push-pull" models [9], or based on dynamical systems [10,11], but never the scenario has been plugged into a statistical-mechanics framework. In this work we want to attempt a first minimal step toward a quantitative approach in this direction.

Disordered statistical mechanics, even though developed in the framework of theoretical physics [12], deals (prevalently) with the equilibrium properties of collections of agents which interact among themselves via competitive

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exchanges [6]: it reveled a surprisingly high capability to manage fields of research far form physics as neurobiology [13], immunology [14], economics [15] and, recently, quantitative sociology [16–21].

The main idea is that when the amount of agents is sufficiently large, disordered statistical mechanics lends support as a stochastic optimization procedure which aims to predict the global behavior of complex systems by considering, in a probabilistic framework, the microscopic deterministic dynamics of all the constituents of the system itself.

Its methodical application for building a "quantitative sociology" has been strongly advocated by social scientists with a background in microeconomics [22].

As a first attempt we propose a simple model for analyzing equilibrium decision making [23] when a population of individuals (*i.e.* a country) experiences interactions with incoming people afferent by several other places all over the world.

As customary within theoretical-physics methodologies, the model has a certain number of "assumptions" that the reader may find "too strong to describe reality". This is a key feature of this approach: dealing with all the details of the phenomenon is surely appealing but more than prohibitive for the complexity involved. Moreover there is the danger of making a *snapshot* of the reality rather than a model with some predictive and descriptive capabilities. On the contrary, if via a minimal model some features can still be retained, this may offer a suggestion for understanding the main degrees of freedom to take into account for future speculations.

In the last section we will show that even (partially) relaxing our simplifying assumptions (that we are going to emphasize here after), the features of our model are retained.

To simplify the mathematical treatment at this first stage, what we postulate is

– When making the host and the migrant communities interact, we neglect the underlying social network (the so-called "small world" [24]) such that each of the N citizens, sooner or later, meet each of the P migrants (mean-field approximation).

– Noticing that the amount of the host agents is huge, many of them will share a negative experience with the migrants while many others will share a positive one; these "exchange interactions" ξ are thought of to be randomly chosen with a centered symmetric probability, *i.e.* $\mathbb{P}(\xi=+1)=\mathbb{P}(\xi=-1)=1/2$.

– Every dependence of the host opinions on the rate of migration is also refused (leaving this task for future improvements of the model) and we consider only the amount of foreigners with respect the host bulk.

– Cultural background (including media influence on the host community) that we encode via an external field h is taken into account to influence uniformly each citizen; models with random or time-dependent fields are again left for future improvements.

While, at least the limits of $P/N \to 0$ (no immigrants scenario) and $P/N \to \infty$ (only immigrants scenario) are clear and poorly interesting as they correspond to absence of social mixtures, in between those two limits complex behaviors may occur.

We obtain two (intertwined) main results:

– First we prove that, when considering our model for interactions between these two parties, this is equivalent (shares the same equilibrium features) to considering a particular Hopfield model of associative neural network [25] inside the host community: In "social terms", when looking at the host country, the population spontaneously develops a "consciousness" of the interactions with the immigrants. Experiences of the migrants are stored into a collective "social memory".

– Interestingly, this happens if the ratio of the amounts of immigrants to the hosts is smaller than a few per cent (depending on the particular choice of the control parameters). If the ratio becomes higher (toward a high multi-ethnic picture) this property falls down (blackout scenario [13]) and there is no memory of the past interactions: the system ages as a canonical spin-glass [6], defining a transition toward an integrated state of the immigrants by the host community.

We stress that our model does not predict any transitions from a hostile attitude toward a benevolent one; instead there is a phase in which the host collective memory is able to formulate opinions on the migrants (these can be either positive or negative or even fluctuating) and there is a phase in which this memory does not work any longer (the immigrants become unaffected by the judgement). In the latter case, they lose their "immigrant" label and indifference with respect to a judgement in this sense is achieved, which we define as *social integration* or better *normalization* in the sense that the foreigners presence has become normal and is no longer noticed.

Social memory of immigrant's interactions. – Let us concretely consider a first population of N citizens $i \in (1, \dots, N)$; each citizen has an opinion $\sigma_i \in \pm 1$, where $+1$ represents a positive attitude with respect to the migrants, viceversa for -1. As this is supposed to be the host population ensemble, no null judgements are allowed for the citizens ($\sigma \neq 0$): each agent is supposed to share a cultural relation with the others and must take a net position with respect to the phenomenon (the diluted case allowing even $\sigma = 0$ will be briefly discussed at the end).

The second population is made by P real agents $\tau_\mu \in \mathbb{R}$, $\mu = 1, \dots, p$. This community is composed by all the incoming immigrants and, to emphasize this lacking of cultural aggregation, its *a priori* probability distribution $\mathbb{P}$ is chosen as a standard centered Gaussian, *i.e.*

$$\mathbb{P}(\sigma_i) = \frac{1}{2}\left(\delta(\sigma_i - 1) + \delta(\sigma_i + 1)\right), \tag{1}$$

$$\mathbb{P}(\tau_\mu) = \frac{1}{\sqrt{2\pi}} \exp\left(-\tau_\mu^2/2\right). \tag{2}$$

Both the communities need to be very large, as, dealing with phase transitions [26,27], we will be interested in making both N and P diverge, such that their ratio is held finite and acts as a control parameter of the theory, *i.e.* $0 < P/N = \alpha \ll 1$ (for example in Italy the amount of citizens is $\sim 60 \cdot 10^6$ and the amount of immigrants $\sim 30 \cdot 10^5$, both are "large numbers" and their ratio defines $\alpha \sim 5 \cdot 10^{-2}$).

Now we encode these interactions into a cost function (a Hamiltonian $H_{N,P}(\sigma, \tau; \xi, h)$, where ξ is quenched and h externally tunable) in a mean-field way, via both positive and negative couplings ξ_i^μ such that

$$H(\sigma, \tau; \xi, h) = -\frac{1}{\alpha^{1/4}\sqrt{N}} \sum_{i,\mu}^{N,P} \sigma_i \xi_\mu^i \tau_\mu - h \sum_i^N \sigma_i, \tag{3}$$

with $\mathbb{P}(\xi_i^\mu = 1) = \mathbb{P}(\xi_i^\mu = -1) = 1/2$, ($\mathbb{P}$ being the ξ probability distribution).

The meaning of the cost function (3) is clear: each i-th citizen meets each μ-th migrant; afterwards he/she experiences a *pressure* by the media and his/her historical background, which influence uniformly the host opinions via the external field h. The success of the meeting is suggested by the value of the ξ_i^μ: if $\xi_i^\mu > 0$ both the i-th citizen and the μ-th migrant will tend to have the same reciprocal judgement (*i.e.* $\sigma_i > 0, \tau_\mu > 0$ or $\sigma_i < 0, \tau_\mu < 0$): it can be positive (for example a low-cost-employee offer which may satisfy both of them) or negative (for example a robbery which may give rise to reciprocal hostility). $\xi_i^\mu < 0$ is also clear: it reflects cases where only one of the two meeting agents is satisfied by the interaction while the other is not.

Overall the set of obtainable scenarios for interactions is complete.

We stress that the single event is not of fundamental importance: we are interested in understanding how the country globally responds to the phenomenon. Further, for a particularly bad situation, given the huge number of agents, there will be probably a particularly good one and even more important a judgement in a single citizen will probably change over the time, but as one may change, even the others, leaving the averaged opinion unaffected by these fluctuations.

The global country judgement on the migrants is defined obviously as $m = N^{-1}\sum_i^N \sigma_i \in \{-1, +1\}$ such that if $m = -1$ a complete antagonist attitude is shown, (and viceversa for $m = +1$), while $m = 0$ stands for indifference.

The square root in the normalization of the Hamiltonian (instead of a power 1), reflects the several cancellations which happen on the volume of the agents due to the conflicting interactions [27].

Once introduced the partition function $Z(\alpha, \beta; \xi, h)$, defined as

$$Z(\alpha, \beta; \xi, h) = \sum_{\{\sigma\}} \int_{-\infty}^{+\infty} \prod_\mu^P \mathrm{d}\tau_\mu e^{-\tau_\mu^2/2} e^{-\beta H(\sigma,\tau;\xi,h)}, \tag{4}$$

at first we stress that another control parameter, β, tunes the strength of the interactions between the two parties (it can also be thought of as the inverse of a noise inside the interaction network [25], reflecting the lacking of a full deterministic behavior as depicted by our assumption). Then, using $\langle\rangle_\xi$ as the average over the quenched ξ distribution, the equilibrium properties of this system are found by studying the related free energy $f(\alpha, \beta, h) = (-1/\sqrt{\alpha}N)\langle \ln Z(\alpha, \beta, \xi, h)\rangle_\xi$.

In a nutshell, at each given triplet (α, β, h), the minima of this free energy display equilibrium between the cost function minimization and severe entropy-related constraints, as, due to the dichotomy of the interactions, the system behaves as a bipartite spin-glass [25].

By applying the Hubbard-Stratonovich lemma [26] on the Gaussian fields τ, we map the partition function $Z(\alpha, \beta; \xi, h)$ to

$$Z(\alpha, \beta; \xi, h) = \sum_{\{\sigma\}} e^{-h\sum_i^N \sigma_i} e^{-\frac{\beta^2}{\sqrt{\alpha}N}\sum_{i,j}^N (\sum_\mu^p \xi_i^\mu \xi_j^\mu)\sigma_i\sigma_j}. \tag{5}$$

A new cost function $\tilde{H}(\sigma; \xi, h)$, in terms of only agents σ interacting with each other, is obtained as

$$\tilde{H}(\sigma; \xi, h) = \frac{-1}{\sqrt{\alpha}N} \sum_{i,j}^{N,N} \left(\sum_\mu^P \xi_i^\mu \xi_j^\mu\right) \sigma_i \sigma_j + h \sum_i^N \sigma_i. \tag{6}$$

This second Hamiltonian (an Hopfield model [28,29]) is *hidden* into the first, it shares with it the same equilibrium behavior (*i.e.* the same value of m) and accounts for *social consciousness*.

What it encodes is simple: if both i-th and j-th citizens had a good (or bad) interaction with respect to the μ-th immigrant ($\xi_i^\mu = \xi_j^\mu$), then, when they meet (*i.e.* through the coupling $\sigma_i\sigma_j$), $\xi_i^\mu \xi_j^\mu$ is positive and they reinforce their opinion on the μ-th immigrant. On the contrary, if i-th has a good interaction with the migrant and j-th a bad one (or viceversa) ($\xi_i^\mu \neq \xi_j^\mu$), the exchange between the two citizens weakens their viewpoints.

To have the global behavior one must then sum over all the citizen couples and all the experiences with the migrants.

We note that the interaction strength within the same party (*i.e.* citizen with citizen) is quadratically stronger than with the immigrants (host community believes much more in the judgement of its elements).

So we see that the model we introduced behaves, only in the space of the citizens, as an associative memory [13] with Hebbian synapses, *i.e.* $J_{ij} = N^{-1}\sum_\mu^p \xi_i^\mu \xi_j^\mu$ [29,30].

The mapping is in fact robust: the N dichotomic host judgements σ_i of the social network reflect the N dichotomic (quiescent or firing) neurons of the neural network.

Further, the dichotomic felt experiences ξ_i^μ in the social network map the dichotomic "learned" experiences in the neural network, and the media influence over the host community h mirrors the threshold for firing [13]; even the power one in the normalization of the two-body interaction in eq. (6) turns out to be the proper one of neural networks [25].

Migrant's integration inside the host community. – This bridge is extremely interesting as it naturally pioneers a quantitative definition of *social integration* for the migrants. In fact a long and deep debate among scientists involved in social research is still going on about the choices of the quantifiers for this phenomenon [31], ranging from averaged hire equivalence between citizens and migrants to the percentage of mixed marriages with respect to those inside a community, as well as several others possible indicators [32].

In our framework, the interactions between the two parties are stored as memories into the dialogues of the host population such that the network's free energy has P minima, each of which corresponding to a given ξ (the gauge symmetry which would suggest $2P$ minima is broken by the field h).

Let us now focus on the memory capacities of the model: starting from a generic point of the 2^N possible states of the system made by the citizens, and propagating through a meaningful Markovian dynamics (*i.e.* a Glauber prescription [13] as the cost function (6) obeys detailed balance [26]), the free energy collapses into the minimum of the closer basin of attraction, which are the P learned patterns or their linear combinations (spurious states).

A convenient order parameter set to check the "retrieval" of the network is the P-vector Mattis magnetization $m^\mu = N^{-1}\sum_i^N \sigma_i \xi_i^\mu$, $\mu \in (1,\dots,P)$ [13], such that, if the control parameters range in the associative memory phase, there will be at least a μ for which $m_\mu \to 1$ after enough time.

Of course increasing P over a threshold, the Hebb matrix will approach a Gaussian for the Central-Limit Theorem (blackout scenario): when this happens the neural networks turns into a spin-glass (a Sherrington-Kirkpatrick model [33,34]), the amount of minima becomes exponentially proportional to the bulk [6] (and not linearly, *i.e.* $P \sim \alpha N$), the basins of attraction become all condensed one into another and retrieval is no longer possible.

So, as time elapses, the amount of migrants increases (as well as the number of host experiences) up to this threshold $\alpha_c(\beta,h)$, that defines the blackout: the transition to a glassy landscape makes retrieval no longer achievable and, in the host social network, it is not possible to formulate a global statement of the phenomenon, *i.e.* $m=0$, $m_\mu = 0$, $\forall \mu \in (1,\dots,P)$: consequently the immigrants are integrated as they can no longer be thought of as "immigrants".

To continue with quantitative results, let us discuss the network still in the zero-noise regime ($\beta \to \infty$) and with an infinitesimal field h that assures only gauge breaking, while for general scenario we refer back to textbooks specialized in neural networks [13,35].

- When $\alpha < 5\%$ there is coexistence of retrieval and spin-glass phases, with retrieval minima lower with respect to the spin-glass ones. $m_\mu \neq 0$ for sparse values of μ and spurious states play a considerable role (see the next section).
- When $5\% < \alpha < 14\%$ there is coexistence of the retrieval and spin-glass phases, with spin-glass minima lower with respect to the retrieval ones. No spurious states exist and only one pattern at a time can be minded.
- When $\alpha > 14\%$ only the spin-glass phase survives. There is no longer any retrieval and the *social consciousness* of the phenomenon disappeared.

The importance of the felt experiences. – Another interesting (and commonly felt) feature of social integration is the "importance of the kind of interaction exchanged". In fact we expect society not to weight minimal interactions (*i.e.* a fugace glance in a bus) but only "concrete" ones (*i.e.* a discussion, a shared dinner and so on). Consequently, if statistical mechanics could be a candidate technique to describe social sciences, this phenomenon should emerge naturally. To test this feature let us now change the distribution of the ξ, namely from eq. (7) to eq. (8)

$$\mathbb{P}(\xi) = \frac{1}{2}\Big(\delta(\xi+1) + \delta(\xi-1)\Big), \tag{7}$$

$$\mathbb{P}(\xi) = \frac{1}{\sqrt{2\pi}}\exp(-\xi^2/2). \tag{8}$$

With the latter, the large amount of interactions have little, negligible, weight in judgment formation of the agents and consequently the retrieval (in the proper region of the parameters) should be much weaker than in the former case. We stress that "universality" (as discussed for the SK model in [36]) is not directly involved as both the distributions share globally the same behavior, but we are interested in the values of the Mattis magnetization alone and not in those of the free energy. Actually this point suggests that other "classifiers" for thermodynamics of complex systems should be introduced (see, for instance, [37] for a discussion on this topics).

We investigated both by numerical simulations and by mathematical techniques that the scenario is indeed correct: from a finite-size scaling in a standard Monte Carlo analysis we found that the retrieval obtained in the case of Gaussian interactions goes to zero as N increases, namely $\langle m_N \rangle_\xi \propto 1/\sqrt{N}$, which is corroborated by an analytical argument, which imposes a strictly zero value for the retrieval, in the thermodynamic limit.

To understand this, we have to solve the "analogical neural network" (the Hopfield model with Gaussian memories [38]) in the replica-symmetric scenario, following [25,39] and checking for its retrieval. To this task we have to paste an external field h into the Hamiltonian, whose strength acts at the same time even as a Lagrange multiplier for the free energy. Concretely we label $m_0 = N^{-1}\sum_i^N \xi_0^i \sigma_i$ as the retrieval observable coupled to the field $h = N^{-1} M \sum_i^N \xi_0^i$: the Lagrange multiplier M is the trial magnetization constraining the retrieval. So we have to enlarge the Guerra interpolating scheme [39], that we briefly outline, by accounting even for the trial magnetization in the free energy.

The idea is quite simple: as we mapped these Hebbian models into bipartite spin-glasses, we can apply the cavity fields (or stochastic perturbations) to both the parties in order to extrapolate the thermodynamic behavior, somehow close to the classical techniques when dealing with a single party. In a nutshell in [39] a trial interpolating free energy has been defined as $\tilde{A}(t)$:

$$\tilde{A}(t) = \frac{\mathbb{E}}{N} \log \sum_\sigma \int \prod_\mu^k \mathrm{d}\mu(z_\mu) \exp\left(\sqrt{t}\frac{\beta}{N}\sum_{i,\mu}^{N,k} \xi_i^\mu \sigma_i z_\mu\right) \times \exp\left(\sqrt{1-t}\left(a\sum_i^N \eta_i \sigma_i + b\sum_\mu^k \tilde{\eta}_\mu z_\mu\right) + c\frac{(1-t)}{2}\sum_\mu^k z_\mu^2\right),$$

where both $\eta, \tilde{\eta}$ are random fields in the sense of stochastic stability [40]. We stress that $t \in (0,1)$ interpolates between $t=0$, where the interpolating quenched pressure is made of non-interacting systems (a series of one-body problems) whose integration is straightforward, and the opposite limit, $t=1$, that recovers the correct quenched free energy.

The plan is then to evaluate the t-streaming of such a quantity and then obtain the correct expression by using the fundamental theorem of calculus:

$$A(\beta) = \tilde{A}(t=1) = \tilde{A}(t=0) + \int_0^1 \mathrm{d}t' \left(\partial_t \tilde{A}(t)\right)_{t=t'}. \quad (9)$$

Now we enlarge this scheme by introducing the magnetization, namely adding the following extra-term to the Boltzmann factor:

$$\exp\left(t\frac{\beta N}{2} m_0^2\right)\exp((1-t)\beta N m_0 M),$$

and compute the streaming with respect to the Lagrange multiplier (which has to be evaluated at $t=0$ to obtain the proper thermodynamic setting).

After some algebra, we arrive to the self-consistent relation

$$\langle m_0 \rangle_\xi = \beta \langle m_0 \rangle_\xi (1 - \langle q \rangle_\xi), \quad (10)$$

where $q \in [-1,+1]$ is the replica-symmetric overlap. It is straightforward to conclude that the only available solution is $\langle m_0 \rangle_\xi = 0$: if the interactions are weak, the social consciousness no longer develops.

It is worth noting that despite Granovetter results [5] suggest the importance of the "weak ties" for the network topology, from our viewpoint only the nodes with strong links do weight in forming global consensus, conferring an even more complex fashion to social behavior: weak ties, however, seem to play a fundamental role in determining the small-world topology when considering only imitative interactions (as it is immediate to check considering the dichotomic Hopfield model with the ξ's defined on $[0,+1]$ instead of $[-1,+1]$).

Toward correlations and small-world properties. – Finally we want to report on some extensions relaxing some of the hypotheses we followed. It is well known in social theory that the mean-field approach we used is too rude. In fact, from the early investigations of Stanley Milgram [41] up to the recent formulations by Watts and Strogatz [24,42], it is known that a proper underlying topology for the interactions is the *small world*. It has been shown by Coolen and coworkers that neural networks on these graphs are very close to the standard one by Hopfield [43,44].

Furthermore the fully uncorrelated experiences maybe a too rude simplification as well. It is in fact very natural to assume that correlations, at least due to similarity among migrants with several common features (*i.e.* religion, political views, lifestyle), do exist. To take into account these features we should introduce a probability distribution for the experiences as

$$P(\xi) = \frac{1}{2}(1+a)\delta(\xi+1) + \frac{1}{2}(1-a)\delta(\xi-1),$$

which naturally enlarges the previous scheme (that is recovered here for $a=0$) as $\langle \xi_i^\mu \xi_i^\nu \rangle = \delta_{\mu\nu} + a^2(1-\delta_{\mu\nu})$. In [28], Amit and coworkers developed a correlated memory neural network in which they showed that upon rescaling the Hebbian synaptic matrix as

$$J_{ij} = \sum_\mu^p (\xi_i^\mu - a)(\xi_j^\mu - a)$$

again the behavior of the network is largely unaffected by the modification, the whole conferring a certain degree of robustness to our minima model.

* * *

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